Influence of Imperfect Cophasing on Performance of MRC Receiver of QPSK Signals Transmitted over Weibull Fading Channel

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Abstract — **This paper presents an analysis of maximum ratio combining (MRC) receiver of quaternary phase-shift keying (QPSK) signals transmitted over a Weibull fading channel. We determine the influence of imperfect cophasing and branches unbalance on bit error rate dependence on average signal-to-noise ratio. The performance dependence on diversity order and fading severity is also observed.**

Keywords — **Cophasing, Diversity systems, Error probability, Fading.**

I. Introduction

In order to reduce the influence of multipath fading on signal detection, it is possible to use spatially separated receiver antennas at the reception and to combine the signals from different branches of the receiver in a certain manner. The optimal combining technique is maximum ratio combining (MRC). This combining technique involves cophasing of the useful signal in all branches, multiplication of the received signal in each branch by the estimated envelope of that particular signal and summing of the received signals from all antennas [1]. By cophasing, all the random phase fluctuations of the signal that emerged during the transmission are eliminated. For this process it is necessary to estimate the phase of the received signal. This phase estimation can be performed from the modulated or unmodulated received signal carrier.

In previous papers concerning this matter the assumptions of the ideal phase estimation of the incoming signal were mainly used [2], [3]. Only in [4] and [5] the influence of the imperfect phase estimation was examined and that was in the case of the equal gain combining (EGC) in the receiver. In paper [4], the influence of the phase error on the error probability in the case of digital binary (BPSK) and quaternary phase-shift keying (QPSK) signal detection was observed. This error probability was calculated applying the Gram-Charlier expansion. The analysis is performed under the assumption of the uncorrelated Rayleigh fading at receiving antennas. In paper [5] closed-form expressions for outage probability and error probability during the BPSK and QPSK signal detection over the Nakagami-*m* fading channel were obtained. Dual branch EGC technique was applied. In both papers the analysis of the system was done under the assumption that the branches of the receiver are balanced. However, as it is well-known [6], it is very rare to meet in practice the identical propagation paths in each branch. Also, the electrical components in different branches of the receiver are not ideal. Thus, it is also of interest to consider the influence of the branch unbalance in the receiver on system performances.

In this paper an analysis of QPSK signal reception is presented. The signal is transmitted over the Weibull fading channel and MRC technique is applied in the receiver, while the receiving branches are assumed unbalanced. The phase estimation is performed from the unmodulated carrier and is not ideal. The difference between the phase of the incoming signal and the estimated phase for that signal is a statistical process and it follows the Tikhonov distribution [4], [5], [7]. It is shown in which measure the imperfect phase estimation affects the bit-error rate (BER).

II. System Analysis

The Weibull distribution is frequently used in fading modeling in urban surroundings, in cases when the Rayleigh distribution is not adequate. This distribution is empirically obtained and it was originally used as a model in system reliability analysis [8], [9]. In [8] it is shown that the Weibull distribution gives a good matching with measurement results of digital enhanced cordless telecommunication (DECT) systems which work on 1.98 GHz. Also, the measurement results on 900 MHz, presented in paper [9], show that this distribution can also be used as a multipath fading model in outdoor surroundings. The fading model with the Weibull

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distribution assumes a signal consisted of multipath clusters in non-homogeneous propagation medium. The resulting envelope is obtained as a function of modulus of a sum of multipath components.

The signal in the i-th receiving antenna (Fig. 1) can be expressed as

$$s_i(t) = r_i(t) e^{j\varphi_i(t)} \cdot A e^{j\phi_n} + n_i(t), \quad i = 1,2,\ldots,L \ , \quad (1)$$

where $r_i(t)$ is a fading envelope and $\varphi_i(t)$ is a random phase fluctuation occurring during the propagation over a fading channel.

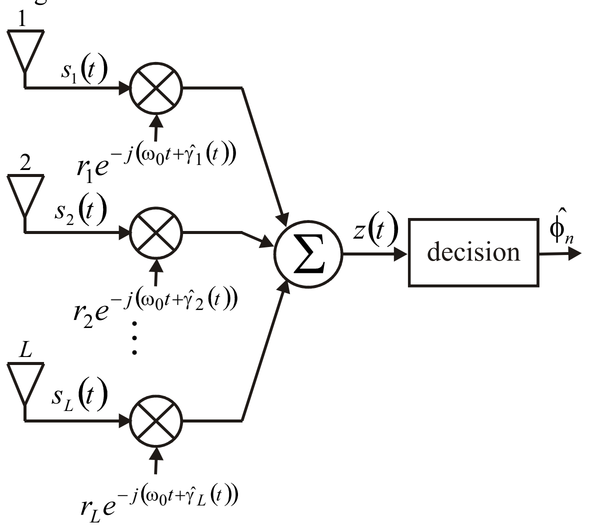

Fig. 1. Model of MRC receiver.

In every branch, fading is frequently non-selective, it doesn't change during one signal and it is independent from symbol to symbol. Also, there is no correlation between the fading in different antennas. A probability density function (pdf) of the fading envelope in the i-th branch of the receiver follows the Weibull distribution [10]

$$p_{r_i}(r_i) = \frac{\alpha_i}{\Omega_i} r_i^{\alpha_i - 1} \cdot \exp\left(-r_i^2 / \Omega_i\right), \quad r_i \geq 0 \qquad (2)$$

where α_i is the fading parameter, and Ω_i=E$\{r_i^{\alpha i}\}$, while E{.} is a mathematical expectation operator. It can be shown that [10]

$$\mathrm{E}\left\{r_i^{\,n}\right\} = \Omega_i^{\,n/\alpha} \Gamma\left(1 + \frac{n}{\alpha_i}\right), \qquad (3)$$

where $\Gamma(.)$ is a Gamma function [11, eq. (8.310/1)]. Amplitude of the useful signal is denoted as A and it can be assumed equal one, without loss of generality. The signal phase, which holds the information about transmitted symbol, is given as ϕ_n. In the case of QPSK signal, it can take one of the following values: {π/4, 3π/4, 5π/4, 7π/4}. If we assume a root mean square (RMS) amplitude equal one

$$\mathrm{E}\left\{r_i^{\,2}\right\} = 1 = \Omega_i^{\,2/\alpha_i} \Gamma\left(1 + \frac{2}{\alpha_i}\right), \qquad (4)$$

then it is

$$\Omega_i = \left(1 / \Gamma\left(1 + \frac{2}{\alpha_i}\right)\right)^{\alpha_i / 2}. \qquad (5)$$

The additive white Gaussian noise (AWGN) with a zero mean value and variance σ_i^2 in the i-th branch is denoted as $n_i(t)$. The standard deviation of this Gaussian noise is presented as

$$\sigma_i = \sqrt{\mathrm{E}\left\{r_i^{\,2}\right\} / \left[2 \log_2 M 10^{\gamma_{bi}} \exp\left(-\delta(i-1)\right)\right]} \ , \quad (6)$$

where M is the number of phase levels, δ is an unbalance coefficient and γ_{bi} is the average bit signal-to-noise ratio (SNR) in the i-th branch of the receiver, given in decibels.

After signal cophasing and multiplying the received signal in each branch by the estimated envelope of that particular signal, resulting signal after the combining is

$$z(t) = \sum_{i=1}^{L} \left(A r_i^{\,2}(t) e^{j\phi_n} e^{j\varphi_i(t)} + r_i n_i(t)\right), \qquad (7)$$

where L is the number of branches at the receiver and $r_i(t)$ is a fading envelope in the i-th branch. The difference between the phase of the received signal in the i-th receiving branch, $\gamma_i(t)$, and the estimated phase in the same branch, $\hat{\gamma}_i(t)$, is denoted as $\varphi_i(t) = \gamma_i(t) - \hat{\gamma}_i(t)$. If the phase estimation is performed from an unmodulated signal carrier and if additive white Gaussian noise is the only present noise in the phase-locked loop (PLL), then the pdf of this phase noise is [4], [5], [7]

$$p_{\varphi_i}(\varphi_i) = \frac{1}{2\pi} \frac{\exp(\varsigma_i \cdot \cos(\varphi_i))}{\mathrm{I}_0(\varsigma_i)}, \quad -\pi < \varphi_i \leq \pi \ , \quad (8)$$

where $\mathrm{I}_0(x)$ represents a modified Bessel function of the first kind and order zero with x argument [11, eq. (8.406)] and ζ_i is the SNR in PLL circuit in the i-th branch of the receiver, which can be expressed as a function of phase error variance $\sigma_{\varphi_i}^{\ 2}$ [4], [5], [7]

$$\zeta_i = 1 / \sigma_{\varphi_i}^{\ 2}. \qquad (9)$$

After the analysis of the receiver behavior and a few mathematical manipulations, one can show that BER in the case of QPSK detection for a diversity system of the second order can be expressed as

$$BER = 0.25 \int_{r_1}\int_{r_2}\int_{\varphi_1}\int_{\varphi_2} \left\{ \mathrm{erfc}\left(\frac{\sum_{i=1}^{2} r_i^2 \cos(\pi/4 - \varphi_i)}{\sqrt{2}\sigma} \right) + \right.$$

$$\left. + \, erfc\left(\frac{\sum_{i=1}^{2} r_i^2 \cos(\pi/4 + \varphi_i)}{\sqrt{2}\sigma} \right) \right\} \times \qquad (10)$$

$$\times \, p_{\varphi_1}(\varphi_1) p_{\varphi_2}(\varphi_2) p_{r_1}(r_1) p_{r_2}(r_2) d\varphi_2 d\varphi_1 dr_2 dr_1$$

where erfc(.) represents a complementary error function [11, eq. (7.1.2.)], $p_{\varphi_1}(\varphi_1)$ and $p_{\varphi_2}(\varphi_2)$ are pdf's of the phase error in the first and second branch, respectively, and $p_{r_1}(r_1)$ and $p_{r_2}(r_2)$ are pdf's of the fading envelope in the first and second branch. It is also worth

$$\sigma = \sqrt{r_1^{\,2}\sigma_1^{\,2} + r_2^{\,2}\sigma_2^{\,2}} \ . \qquad (11)$$

In a similar manner BER expressions for a higher diversity order can be obtained.

III. Numerical Results

Numerical results are obtained in two different ways – using numerical integration and applying Monte Carlo simulations. In order to obtain BER values, it is necessary to perform numerical integration in (10). As one can see, if there is a MRC receiver with two branches, a fourfold numerical integration appears. This numerical integration is performed using Gaussian quadrature formulas. The number of points was increased until the predetermined accuracy is reached. In Monte Carlo simulations 2^{31}-1 symbols were used maximally. The minimal number of symbols that were used for the BER value estimation was 10^4 and criterion for the loop termination is 4000 errors. In order to present the obtained results in a more convenient way, we assumed parameters of the Weibull fading and phase error standard deviations equal in all branches: $\alpha_1=\alpha_2=\ldots=\alpha_L$, $\sigma_{\varphi_1}=\sigma_{\varphi_2}=\ldots=\sigma_{\varphi_L}$. The average bit SNR in the first branch of the receiver is denoted as $\gamma_{b_1}=\gamma_b$.

In Fig. 2 the influence of the phase error standard deviation on the BER values in the case of QPSK detection is presented. For moderate values of γ_b, with the increase of γ_b, BER values decrease. For large values of γ_b, with the increase of γ_b, BER remains constant. This BER floor, which appears, depends on the phase error standard deviation. For example, if σ_φ rises from 12.5° to 20°, BER floor changes from $2.8\cdot10^{-5}$ to $4.5\cdot10^{-3}$.

The influence of the phase error standard deviation on the BER values is more obvious in Fig. 3. One can notice that the QPSK modulation format is much sensitive to the phase error standard deviation fluctuations. BER values start to grow beginning with the σ_φ value of about 8°.

In Fig. 4 the influence of the unbalance factor on system performances is shown. For small and moderate values of γ_b, unbalance factor δ significantly affects BER values. For example, in order to achieve the BER value 10^{-3}, for σ_φ=15°, it is necessary to have γ_b=12.8 dB in the case of a perfectly balanced receiver and γ_b =19.2 dB for δ=2. For large γ_b values, the BER floor appears and its value does not depend on δ - BER floors overlap. For σ_φ=15° BER floor is $3.2\cdot10^{-4}$.

In Fig. 5 the influence of the Weibull fading parameter on the BER values in the case of QPSK detection is presented. It can be observed that the values of BER floor which appears due to imperfect carrier signal extraction, depend on the fading parameter. If we wanted to obtain BER value 10^{-5}, we would need γ_b=12.5 dB, in the case of α=4, and γ_b=20.5 dB, in the case of α=2 (larger fading severity than in the previous case).

In Fig. 6 the influence of the number of receiving antennas on system performances is given. Power gain is the highest when the order of diversity system changes from L=1 to L=2 and the increasing of number of diversity branches reduces the additional gain. BER value of 10^{-5} is not possible to obtain with the number of branches lower than four, because with the diversity of the first, second and third order a BER floor larger than 10^{-5} appears.

In this paper we achieved very good matching between the numerical results, obtained using numerical integration, and results, obtained by Monte Carlo simulations, due to very rigorous criteria for BER value estimation, which were previously explained in detail.

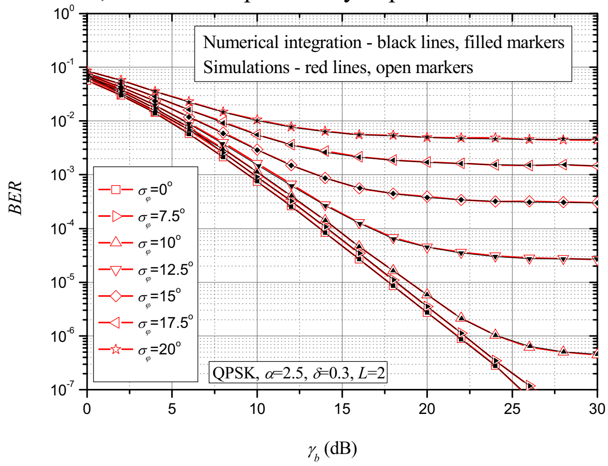

Fig. 2. Dependence of BER on average bit SNR for different values of phase error standard deviation.

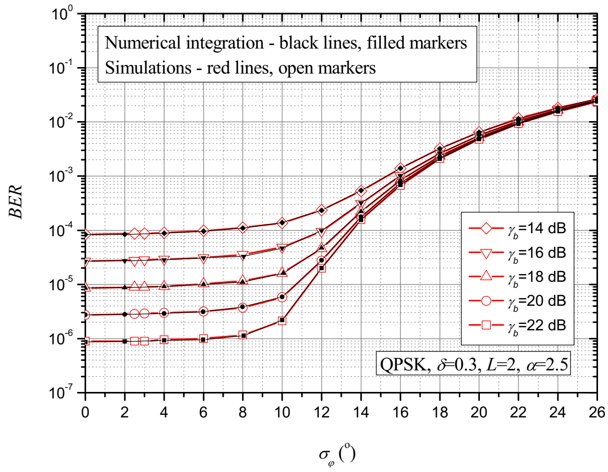

Fig. 3. Dependence of BER on phase error standard deviation for different values of average bit SNR.

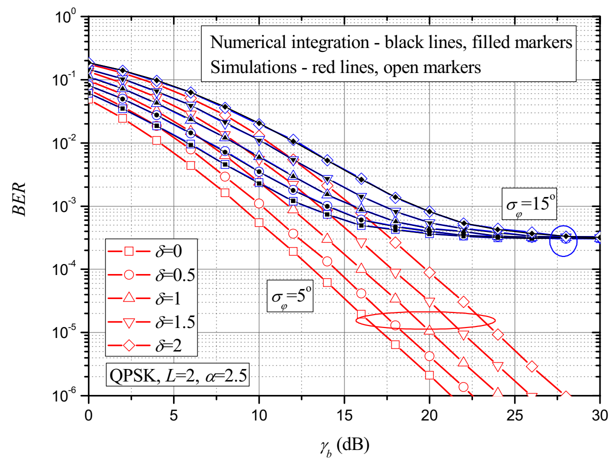

Fig. 4. Dependence of BER on average bit SNR for different values of unbalance factor.

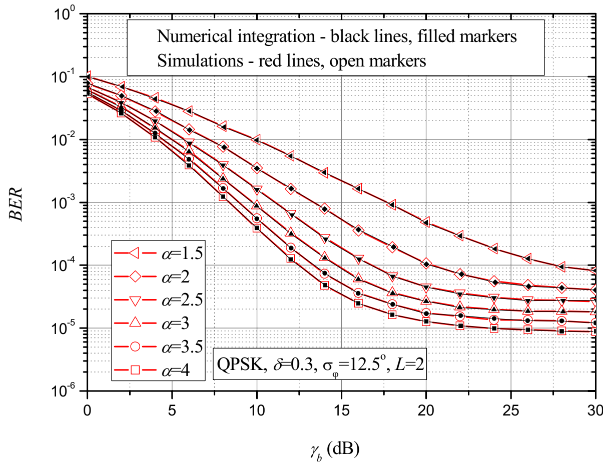

Fig. 5. Dependence of BER on average bit SNR for different values of the Weibull fading parameter.

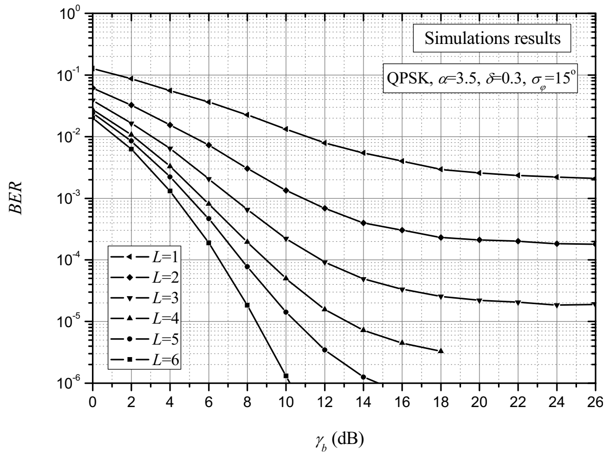

Fig. 6. Dependence of BER on average bit SNR for different values of receiver diversity order.

IV. Conclusion

In this paper a relation between the phase error standard deviation and BER in a MRC receiver with unbalanced branches, while QPSK signal over the Weibull fading channel is being detected, has been established. Using presented results and procedure that was elaborated here, it is possible to calculate the necessary value for phase error standard deviation under the condition that a predetermined BER value has not been exceeded. Based on that, it is possible to optimize the circuit for the phase estimation of incoming signal, so that this calculated value for a phase error standard deviation will not be exceeded.

It is shown that random phase error fluctuations cause a BER floor (Fig. 2 and Fig. 3). It is confirmed that the values of phase error standard deviation significantly affect the values of this BER floor. Results, presented in Fig. 3, show in which measure QPSK signal detection is sensitive to the influence of the imperfect phase error estimation of the incoming signal. As it is already mentioned, BER values are approximately the same until the phase error standard deviation reaches the value of about σ_φ=8°. It illustrated in which range of BER values the unbalance of the receiver has a significant role (Fig. 4). It is shown in which measure fading severity, combined with the imperfect phase estimation, affects BER values (Fig. 5). Results in Fig. 6 enable one to find a compromise between the transmission quality (BER value) and a system complexity (number of branches at the receiver).

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