Analysis of Electromagnetic Scatterers Using Hybrid Higher Order FEM-MoM Technique

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Abstract **— A higher order large-domain hybrid finite element – method of moments (FEM-MoM) technique is presented. The discretization and solving FEM and MoM parts of the problem and the coupling of the two methods are described. A benchmark example of analysis of a scatterer with pronounced curvature at resonant frequency is given, demonstrating accuracy and robustness of the presented hybrid technique.**

Keywords **— Electromagnetic analysis, finite element methods, hybrid techniques, method of moments.**

I. Introduction

HYBRID finite element-boundary integral (FE-BI) techniques have been popular for the exact truncation of the unbounded spatial domain in the finite element method (FEM) analysis for quite some time [1]-[4]. They basically divide the problem into an interior and exterior region. The field in the interior, usually inhomogeneous, region is expressed using FEM and the field in the exterior, homogeneous, region is represented by some sort of a boundary-integral equation (BIE). The fields in the interior and exterior regions are then coupled by the field continuity conditions at the FE-BI boundary surface. Good properties of both FEM, in dealing with inhomogeneities in the bounded domain, and BIE, in dealing with homogeneous open domains, methods are thus united into a powerful hybrid technique. Since the BIE computational methodologies correspond to solutions of surface integral equations (SIEs) based on the method of moments (MoM), the hybrid methods are sometimes also referred to as FEM-MoM techniques.

A tremendous amount of effort has been invested in the research of the FE-BI techniques in the past two decades. This led to many improvements of the original ideas and some novel techniques have been developed. A simple and effective hybridization concept is given in [5], followed by the review of contemporary hybrid techniques given in [6] and the study of a variety of FE-BI formulations for three-dimensional (3-D) electromagnetic scattering by inhomogeneous objects and development of a novel technique immune to the problem of interior resonances, which also employed the fast multipole algorithm (MLFMA) for efficient solving of the BIE matrix [7]. A notable novelty is introduced in [8], where a hybrid FEM-MoM formulation, which leads to a symmetric FE-BI matrix, has been derived and generalized in the symmetric hybrid formulation in [9].

Although a few of the most recent works include a development of hybrid FE-BI techniques with higher order basis functions [10]-[12], none of the presented methods seem to employ higher order BI formulation in the hybrid scheme and none of the techniques fully exploits the potential of the higher order field-expansion – the highest order of the basis functions presented in the given solutions is three.

Our goal in the present work is to demonstrate the accuracy and efficiency of our novel 3-D fully higher order FEM-MoM method developed by hybridizing our existing higher order FEM [13]-[14] and MoM [15] techniques. In our analysis method we emphasize large-domain geometrical modeling by using large curvilinear Lagrange-type volume and surface elements in coarse meshes and hierarchical polynomial vector basis functions of high (arbitrary) orders. In this way, elements of different shapes and sizes and different orders of polynomial field- and current-approximations can be used in the same FEM-MoM mesh. Thus we fully exploit the potential of the higher-order modeling which can lead to the reduction in computation costs by one to two orders of magnitude when compared to low-order (small-domain) techniques, for the same or better accuracy [13]-[15]. Additionally, the new hybrid method can incorporate multiple MoM objects and FEM regions in a global unbounded MoM domain. Thus our method is not strictly dependent on the FE-BI scheme; it is versatile in a sense that FEM regions can, but not necessarily have to, exist in the overall MoM environment.

In section II we present the theoretical background of a higher order hybrid FEM-MoM technique for analysis and design of antennas and microwave devices. In section III we give an example of analysis of electromagnetic

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scatterer with pronounced curvature at the internal resonance frequency by the presented hybrid FEM-MoM technique.

II. Theoretical Background

In our hybrid higher order FEM-MoM technique, the solution in the exterior region is obtained utilizing higher order MoM for discretizing the set of coupled electric/magnetic field integral equations (EFIE/MFIE) with electric and magnetic surface currents as unknowns. The solution in the interior region of the problem is obtained utilizing higher order FEM for discretizing the curl-curl electric-field vector wave equation. The two regions are denoted as region *a* (exterior or MoM region) and region *b* (interior or FEM region), as shown in Fig. 1. The two methods are coupled at the boundary of the interior (FEM) region via MoM electric surface currents, which are directly related to the boundary conditions required for closing of the FEM region. The FEM-MoM boundary can be moved some distance away from the actual objects within the FEM domain (e.g., when the objects are metallic), or it can coincide with the object boundary surface (e.g., for dielectric objects).

Theoretically, multiple MoM objects and multiple FEM regions can be present in an overall MoM environment. In this arrangement, for example, piecewise homogeneous dielectric domains can be modeled as MoM objects (via surface equivalence theorem) or as FEM regions. Metallic objects, on the other hand, can be modeled as MoM objects (via surface electric currents) or they can be enclosed in the virtual dielectric domain and treated as FEM regions.

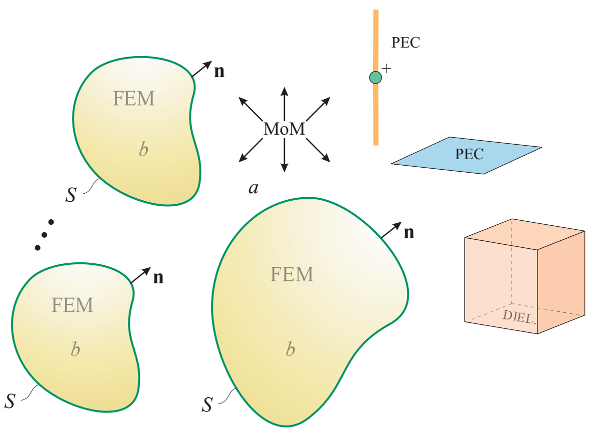

Fig. 1. Decomposition of an electromagnetic structure into a MoM (exterior) and a FEM (interior) region, denoted as regions *a* and *b*, respectively.

The versatility and flexibility of this concept, combined with the higher order large-domain modeling framework, allows for efficient modeling of complex EM structures provided that basic geometrical modeling units, surface and volume elements, can be large and flexible. Hence, unique generalized curvilinear quadrilateral surface elements and hexahedral volume elements of arbitrary geometrical orders are employed for the tessellation of MoM and FEM regions, respectively. A generalized curvilinear quadrilateral, like the one shown in Fig. 2(a), has been chosen for surface geometrical modeling in MoM/SIE. Similarly, the element type used for volume geometrical modeling in the interior (FEM) region is the generalized curvilinear hexahedron, shown in Fig. 2(b). Lagrange interpolating polynomials [13]-[15] are used to define mappings from the square/cubical parent domains to the Lagrange-type curved quadrilateral/hexahedron. Finally, polynomials of arbitrary orders are used in a hierarchical fashion in the construction of vector basis functions to approximate unknown currents and fields.

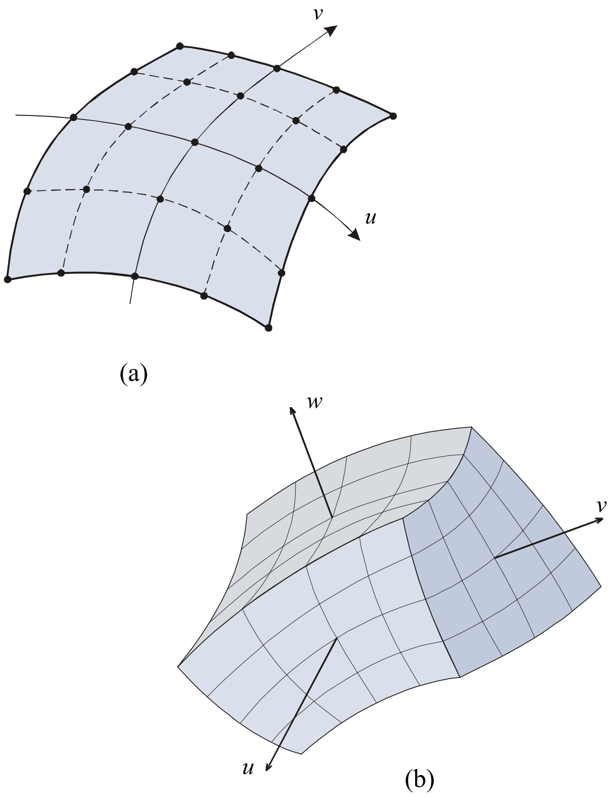

Fig. 2. Lagrange-type curved parametric elements for geometrical modeling: (a) quadrilateral, and (b) hexahedron.

Basic field equations in the domains *a* and *b* are:

$$\mathbf{E}^a = \mathbf{E}_\mathrm{J}(\mathbf{J}_\mathrm{S}) + \mathbf{E}_\mathrm{M}(\mathbf{M}_\mathrm{S}) + \mathbf{E}^\mathrm{inc}, \quad \mathbf{E}^b = \mathbf{E}^b(\mathbf{J}_\mathrm{S}), \quad (1)$$

and

$$\mathbf{H}^a = \mathbf{H}_\mathrm{J}(\mathbf{J}_\mathrm{S}) + \mathbf{H}_\mathrm{M}(\mathbf{M}_\mathrm{S}) + \mathbf{H}^\mathrm{inc}, \quad \mathbf{H}^b = \mathbf{H}^b(\mathbf{E}^b), \quad (2)$$

where $\mathbf{J}_\mathrm{S}$ and $\mathbf{M}_\mathrm{S}$ represent electric and magnetic surface current vectors, respectively.

The above equations are coupled through boundary conditions at the interior (FEM) region boundary

$$\mathbf{E}^a{}_\mathrm{tan} = \mathbf{E}^b{}_\mathrm{tan} = \mathbf{n} \times \mathbf{M}_\mathrm{S}, \quad (3)$$

$$\mathbf{H}^a{}_\mathrm{tan} = \mathbf{H}^b{}_\mathrm{tan} = \mathbf{J}_\mathrm{S} \times \mathbf{n}, \quad (4)$$

where $\mathbf{n}$ is the outward looking unit normal, to give the resulting hybrid system of equations:

$$-\mathbf{E}_\mathrm{J}(\mathbf{J}_\mathrm{S})_\mathrm{tan} - \mathbf{E}_\mathrm{M}(\mathbf{M}_\mathrm{S})_\mathrm{tan} + \mathbf{E}^b{}_\mathrm{tan} = \mathbf{E}^\mathrm{inc}{}_\mathrm{tan}, \quad (5)$$

$$-\mathbf{H}_{\mathrm{J}}(\mathbf{J}_{\mathrm{S}})_{\tan}-\mathbf{H}_{\mathrm{M}}(\mathbf{M}_{\mathrm{S}})_{\tan}+\mathbf{J}_{\mathrm{S}}\times\mathbf{n}=\mathbf{H}^{\mathrm{inc}}{}_{\tan}\,. \quad (6)$$

Formal current- and field-expansions that are used for the approximation of unknowns in the domains a and b can be represented as:

$$\mathbf{J}_{\mathrm{S}}=\sum_{j=1}^{N_{\mathrm{MOM}}}\alpha_j\,\mathbf{j}_{\mathrm{S}_j}\,,\quad \mathbf{M}_{\mathrm{S}}=\sum_{j=1}^{N_{\mathrm{MOM}}}\beta_j\,\mathbf{j}_{\mathrm{S}_j}\,,\text{ and} \quad (7)$$

$$\mathbf{E}^b=\sum_{l=1}^{N_{\mathrm{FEM}}}\gamma_l\,\mathbf{e}_l\,. \quad (8)$$

where $\mathbf{j}_{\mathrm{S}j}$ are the divergence-conforming hierarchical polynomial basis functions [15], $\mathbf{e}_l$ are the curl-conforming hierarchical polynomial basis functions [13], α_j and β_j are unknown current-distribution coefficients, and γ_l are unknown field-distribution coefficients.

Scattered electric field $\mathbf{E}$ in region a is represented by
$\mathbf{E}=\mathbf{E}_{\mathrm{J}}(\mathbf{J}_{\mathrm{S}})+\mathbf{E}_{\mathrm{M}}(\mathbf{M}_{\mathrm{S}})$,

$$\mathbf{E}_{\mathrm{J}}(\mathbf{J}_{\mathrm{S}})=-\mathrm{j}\omega\mathbf{A}-\nabla\phi\,,\quad \mathbf{E}_{\mathrm{M}}(\mathbf{M}_{\mathrm{S}})=-\frac{1}{\varepsilon}\nabla\times\mathbf{F}\,, \quad (9)$$

and the scattered magnetic field $\mathbf{H}$ in the same region is represented by
$\mathbf{H}=\mathbf{H}_{\mathrm{M}}(\mathbf{M}_{\mathrm{S}})+\mathbf{H}_{\mathrm{J}}(\mathbf{J}_{\mathrm{S}})$,

$$\mathbf{H}_{\mathrm{M}}(\mathbf{M}_{\mathrm{S}})=-\mathrm{j}\omega\mathbf{F}-\nabla U\,,\quad \mathbf{H}_{\mathrm{J}}(\mathbf{J}_{\mathrm{S}})=\frac{1}{\mu}\nabla\times\mathbf{A}\,. \quad (10)$$

Equations (9)-(10) are discretized by the substitution of the formal current-expansions (7) into the potential equations

$$\mathbf{A}=\mu\int_S\mathbf{J}_{\mathrm{S}}g\,\mathrm{d}S\,,\qquad \mathbf{F}=\varepsilon\int_S\mathbf{M}_{\mathrm{S}}g\,\mathrm{d}S\,, \quad (11)$$

$$\Phi=\frac{\mathrm{j}}{\omega\varepsilon}\int_S\nabla_{\mathrm{S}}\cdot\mathbf{J}_{\mathrm{S}}g\,\mathrm{d}S\,,\quad U=\frac{\mathrm{j}}{\omega\mu}\int_S\nabla_{\mathrm{S}}\cdot\mathbf{M}_{\mathrm{S}}g\,\mathrm{d}S\,. \quad (12)$$

In the above potential equations, g is the Green's function to be evaluated separately for each of the homogeneous regions of the external domain. It is defined by

$$g=\frac{\mathrm{e}^{-\gamma R}}{4\pi R}\,,\ \gamma=\mathrm{j}\omega\sqrt{\varepsilon\mu}\,, \quad (13)$$

where ε and μ are permittivity and permeability of the medium, respectively, and ω is the angular frequency of the implied time-harmonic variation.

For region b, the field-expansion (8) is substituted in the curl-curl electric-field vector wave equation

$$\nabla\times\mu_{\mathrm{r}}^{-1}\nabla\times\mathbf{E}-k_0^2\varepsilon_{\mathrm{r}}\mathbf{E}=0 \quad (14)$$

where ε_{r} and μ_{r} are complex relative permittivity and permeability of the inhomogeneous (possibly lossy) internal medium, respectively, and $k_0=\omega\sqrt{\varepsilon_0\mu_0}$ is the free-space wave number.

Galerkin-type solutions for both unknown current distribution coefficients, α_j and β_j, in region a and field distribution coefficients, γ_l, in region b are explained in [15] and [13], respectively, whereas the details of the hybridization process are given in [16].

III. Results and Discussion

Consider a dielectric coated perfect electric conductor (PEC) spherical scatterer shown in Fig. 3. Relative permittivity and permeability of the dielectric are $\varepsilon_{\mathrm{r}}=4$ and $\mu_{\mathrm{r}}=1$, respectively. The radii of the spheres are $a=0.3423\,\lambda_0$ for the PEC sphere and $b=0.4440\lambda_0$ for the dielectric coating, λ_0 being the free-space wavelength. This corresponds to the frequency of the internal resonance of the sphere, thereby introducing significant strain on the low-order techniques which employ fragile iterative sparse solvers [9].

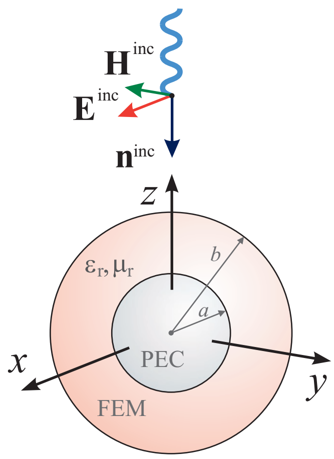

Fig. 3. Scattering by a dielectric coated PEC spherical scatterer ($a=0.3423\,\lambda_0$, $b=0.4440\lambda_0$, $\varepsilon_{\mathrm{r}}=4$, and $\mu_{\mathrm{r}}=1$).

The scatterer, being an example of a structure with pronounced curvature, is modeled using a simple volume geometrical mesh consisting of only 6 curvilinear hexahedral FEM elements of the second geometrical order with 6 attached curvilinear quadrilateral MoM patches of the second geometrical order on the outer boundary. Cross section of this efficient model is shown in Fig. 4.

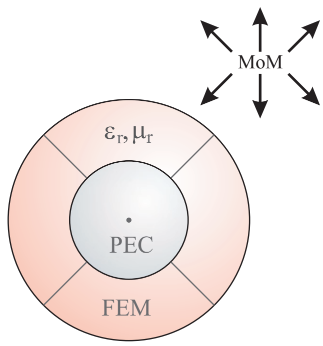

Fig. 4. Cross section of the FEM-MoM model of the spherical scatterer in Fig. 3 consisting of 6 FEM elements with 6 attached quadrilateral MoM patches.

Shown in Fig. 5 is the normalized bistatic radar cross section RCS/λ_0^2 of the coated spherical scatterer for the plane wave incidence indicated in Fig. 3. The adopted field and current approximation orders in the FEM-MoM model are 7 and 6, respectively. The results are compared with the exact Mie's solution and with numerical results obtained by a low-order symmetric FEM-IE [9]. A good agreement of the three sets of results is observed, with the higher order model providing a more accurate RCS prediction at angles around 40 degrees than the low-order FEM-IE model.

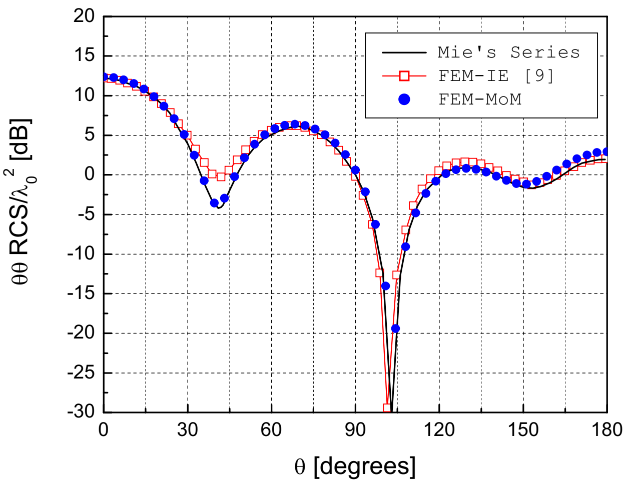

Fig. 5. Normalized bistatic RCS of the coated dielectric sphere from Fig. 3. Comparison of the higher order FEM-MoM solution with the analytic solution and a low-order FEM-IE solution from [9].

IV. Conclusion

A higher order hybrid FEM-MoM technique for analysis and design of antennas and microwave devices and systems has been presented. It combines the advantages and avoids the drawback of each of the methods alone, FEM and MoM, and adds to that the flexibility and efficiency of the higher order techniques. An example of analysis of electromagnetic scatterer with pronounced curvature at internal resonance has been given to show the accuracy and robustness of the presented technique in tackling the open region problems, which are especially problematic when FEM alone is used. Our future work will include the development of effective large-domain hexahedral meshing techniques which will enable modeling of more complex real-world structures.

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