Network Topology Availability Analysis

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Abstract — **In this paper the availabilities of some network topologies are analyzed and calculated. Software based on exact All-terminal Graph reduction Algorithm is developed and applied. Results, conclusions and solutions are presented, demonstrated and discussed.**

Keywords —**availability, network, topology.**

I. INTRODUCTION

THE availability theory is a very interesting research area which has applications in many scientific disciplines, at different levels. In this paper, the availability theory will be applied to the network core whose well-functioning is a major prerequisite for the successful operation of the entire network. Today, more than ever, there is a need for high availability in various types of telecommunication networks at the level of communication devices, but also in the network as a whole. Availability is an expected average time in which a single device or the entire network operates properly. Modern network providers, operators and manufacturers aim for their network and devices to achieve 99.999% availability ("5 nine" availability) which means 5 minutes of failure during the year. They expect to receive an answer on how to design the network (or the device) so that it has a high availability with as low as possible investments. Also, providers are aware that their future users will not forgive any loss of service and because of that, they want their networks to provide continuous service.

The term availability is used for reparable systems and is defined as the probability that the system is working properly in the given moment of time t. As a reminder, reliability is defined as the probability that in the time interval between 0 and t there is no fault in the system. The availability is denoted by $A(t)$ and represents the union of events:

$$A(t) = P(\text{no fault between 0 and } t +$$
$$1 \text{ fault and 1 repair between 0 and } t + \qquad (1)$$
$$2 \text{ faults and 2 repairs between 0 and } t + ...).$$

Because the events in the previous equation are mutually exclusive, equation (1) becomes the sum of the probabilities:

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$$A(t) = P(\text{no fault between 0 and } t) +$$
$$P(1 \text{ fault and 1 repair between 0 and } t) + \qquad (2)$$
$$P(2 \text{ faults and 2 repairs between 0 and } t) +...$$

which is described in detail in [1].

To clarify:
- The first expression in equation (2) is reliability, $R(t)$
- $A(t) = R(t) = 1$ at time $t = 0$
- For $t > 0$, $A(t) > R(t)$
- $R(t) = 0$ when $t \rightarrow \infty$
- As shown in [1], $A(t)=Ass$ when $t \rightarrow \infty$, so as long as repair is possible then $A_{ss} > 0$, where Ass is asymptotic value for availability
- Availability of a network node can be calculated as follows:

$$A_{ss} = \frac{\mu}{\mu + \lambda} \qquad (3)$$

where μ is intensity of repairs per hour while λ is intensity of failures per hour
- Availability of a network link can be calculated as follows:

$$A_{ss} = \frac{\mu}{\mu + \lambda * L / 100} \qquad (4)$$

where μ is intensity of repairs per hour, λ is intensity of failures per hour on 100 km and L is optical link length. These equations are used in this paper for availability calculation of network elements. The same expressions are used in [2] also.

II. METRICS FOR ASSESSING THE AVAILABILITY OF NETWORK TOPOLOGY

The availability of the network should describe the extent to which the network is able to provide us with an appropriate level of continuous operation. To describe the availability values that are used in the research field, but also for practical purposes, the following metrics are defined: two-terminal availability (2TA), k-terminal availability (kTA), all-terminal availability (ATA), and all-operating node availability (AoTA). Each of these metrics is defined as follows:
- Two-terminal availability (2TA) - The probability that there is a path that connects a defined start and termination node. In other words, a single component of a given graph G that describes the network includes both starting and terminating node.

    $Ast = P$ (that nodes s and t are connected)
- K-terminal availability (kTA) - The probability that there is a path between at least k defined nodes. In other words, in a graph G that describes the network, there is a component that connects at least k nodes defined.

Ast = **P** (that the selected k or more nodes are connected)

- All-terminal availability (ATA) - The probability that all nodes are connected. In other words, there is a path between any two nodes in a graph G, which describes the network.
- All-operating terminal availability (AoTA) - Similar to the ATA, this is the probability that all operative nodes in the network are connected. In other words, in a graph G that describes the network, there is a component that contains all nodes which are in operational state. This metric and its value can vary and there are several different interpretations of the situation that one node is operational, but is not connected, because the node through which it was connected is no longer in operational state. This metric does not make sense if the nodes are viewed as ideal.

Which metric is optimal to choose, i.e. what the best metric is in the given case, typically depends on the situation. Two-terminal availability should be used in situations where the network is designed primarily for two nodes: the source and destination of traffic that will process the information obtained. Other nodes in the network have only functionality to forward data to the destination and are called relay nodes. A good example may be several types of wireless sensor networks, and mobile wireless ad-hoc networks. So, in these cases, the controller for receiving and processing information and measuring node represent the termination node and the source node, respectively, during the two-terminal analysis. K-terminal is helpful when there are $k < n$ nodes, for which the network seeks to ensure mutual connectivity. In this way, it is possible to introduce slightly less strict criteria in terms of availability. One good example of when to use k-terminal are military mobile wireless networks, when at some point several nodes for some reason were dropped out of the fight, they may be excluded from these k nodes that are of interest and they can become ordinary relay nodes, [3]. All-terminal is the most stringent criterion and is used when there are extremely high demands for availability. Such a sharp criterion could be a very big problem for designers in the future, when it is expected that the networks would have a very large number of nodes. This will be discussed in the chapter where the obtained results will be analyzed. All operating terminal (as opposed to the all-terminal) metric takes into account continued network operation after the failure of a node, or more precisely, takes into account the limited remaining network function, i.e. the remaining operational network at a certain level.

I. ALGORITHMS FOR AVAILABILITY CALCULATION

For modeling the network topology, directed graphs (Fig. 1) are used sometimes, while more often the availability of the link is the same in both directions. Therefore, the simpler undirected graphs can be used for modeling (Fig. 2), which are also used during the calculation process in this paper.
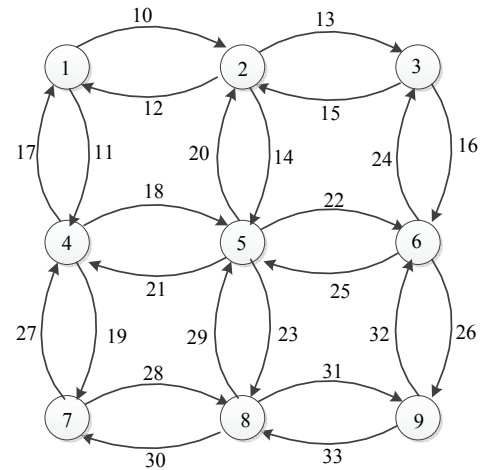

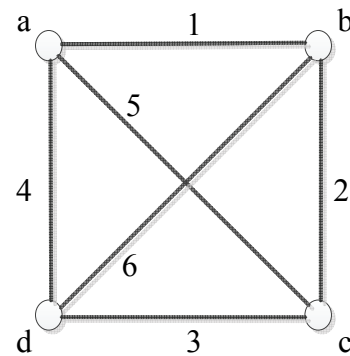Fig. 1. Network topology modeled by direct graph.

Fig. 2. Network topology modeled by undirect graph.

In order to calculate the availability, different algorithms can be used. In English terminology, this calculation is often called network analyzing. Some algorithms are exact (State-Space Algorithm, Graph-reduction algorithm), while the others are approximative and usually estimate upper and lower bounds. These are Equvivalent Link Algorithm, Cut-Set algorithm, Tie-set algorithm. Details can be viewed in [1], [4] and [5]. In the following paragraph, a commonly recommended exact algorithm for computing all-terminals availability - All-terminal Graph-reduction algorithm will be briefly described. This is a recursive algorithm, based on graph transformation and reduction, rather difficult to implement and has an exponential running time.

Before we proceed with the transformation of the graph, which is used for modeling of the network in case of the all-terminal availability calculation, two nodes have to be selected, s and t (start and target). Making transformations is needed until the network is brought to the so-called st branch. Of course, every time when serial and parallel connections are replaced, a single branch within a network is removed and one network transforms to two new ones. Hence, from one network actually two new ones are created. The first one is a network where the branch on which we perform the dismantling works and the other one is a network where that branch does not work. The algorithm ends when each branch of recursion is brought to the equivalent st branch. The availability of every

possible network topology can be calculated using three transformations shown in Fig. 3. When it comes to all-terminal case, serial transformation has the additional term in the denominator $[1 - P(1') P(2')]$, which equals the probability that a node that was excluded in serial transformation is still connected to the network. Other transformations are the same as in the two-terminal case [1], [6] and [7]. Furthermore, in case of all-terminal metrics, when the process of transformation is complete, the resulting probability must be multiplied with the probability that all nodes that are removed during transformation are still connected to the network, [1] and [7].
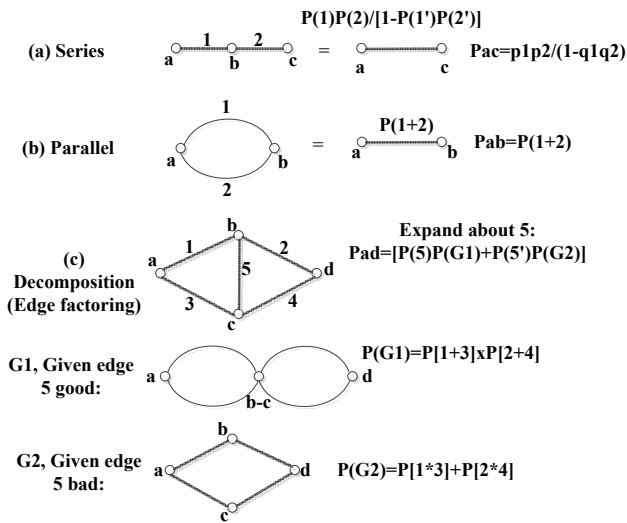

Fig. 3. All-terminal Graph-reduction Algorithm transformations.

II. Software

One segment of the software, which was developed for the future research needs, will be used in this paper for the purpose of testing the topologies of „Telecom Serbia South Network" [2]. The software consists of several modules that are implemented in programming languages Java and C++. The main advantage of this software is its modularity and the ability to test any network topology without any change in the source code, which cannot be achieved by using Markov chains, as there every new network topology carries with itself a new system of differential equations. Using this approach is very important for future research. Software developed in programming language C++ can automatically call the module twoall.1.3.jar implemented in Java and thus, an excellent flexibility in testing is provided.

This paper is focused on the use of twoall.1.3.jar module for calculating all-terminal availability and, therefore, it will be used to test topologies of „Telecom Serbia South Network".

III. Results and Disscusions

In this chapter the focus will be on calculation of the *All-terminal* availability with a practical example of testing topologies given in paper [2]. In that paper, the availability is defined as follows:

The availability of network connection is a possibility of packet transmission between any two network routers.

Furthermore, the paper refers to [8] and [9] where the availability is defined as follows:

It is accepted that the system is correct if there is a connection between TC center with every other TC center and if every TC center works, [8].

Based on the previous, it is obvious that the calculated availability in paper [2] is *All-terminal* availability because the network availability is defined as the probability of each two network nodes connectivity. Moreover, from [2] it can also be concluded that calculated availability is availability at the service level due to mentioning the mechanisms necessary to achieve a high-level quality of service (mechanisms of network recovery, guaranteeing bandwidth and avoiding of drastic delay increase after redirection).

A. Mathematical analysis

In this calculation it is assumed that the protocols' convergence time is always 0 (an automatic traffic redirection), the bandwidth is always sufficient and delay parameter is always satisfied also. Of course, in practice this has never happened and this fact is in favor of that the availability at service level is less than the calculated one. Based on the parameters given in paper [2], availability of all network elements individually can be calculated using equations (3) and (4). All-terminal network availability is calculated as follows:

$$A_{all} = \left(\prod_{i=1}^{n} An(i) \right) * A_{all} \left(Network \Big/ perfectNodes \right) \quad (5)$$

The first term of formula (5) is obtained by the multiplication of all nodes availability, and the second is the availability of network with ideal nodes and real links which are calculated by twoall.1.3.jar software using models given in [1], [5] and [6].

Mathematical proof of equation (5):

Network availability can be calculated by decomposition method. In this method the network is decomposed into two sub-networks as follows:

$$A_{all} = A(R1) * A_{all}(Network/R1) +$$
$$A(\cancel{R1}) * A_{all}(Network/\cancel{R1}) \quad (6)$$

where $A(R1)$ is availability of node $R1$, while $A(\cancel{R1}) = 1 - A(R1)$ is its unavailability.

Using the fact that $A_{all}(Network/\cancel{R1}) = 0$ because node $R1$ does not work in that case, we can obtain formula (5) by repetition of decomposition (6) on all network nodes.

As can be seen, equation (5) consists of two terms:

$$\left(\prod_{i=1}^{n} An(i)\right) \qquad (7)$$

and

$$A_{all}\left(Network\Big/perfectNodes\right). \qquad (8)$$

It can be observed from the results given in the remainder of this chapter that expression (7) has the main role in total availability. If expression (7) is decomposed, it will be evident that in the "*Telecom Serbia South Network*" one network node has the same impact on total availability result as network topology with perfect nodes and real links. Now, the results obtained by twoall.1.3.jar software will be presented and discussed. During those simulations, the same values for λ and μ as in [2] are used for the nodes and links availabilities.

B. Obtained results

The software twoall.1.3.jar has calculated that if the nodes in the network were ideal, topologies from the paper [2] would have the following availabilities:

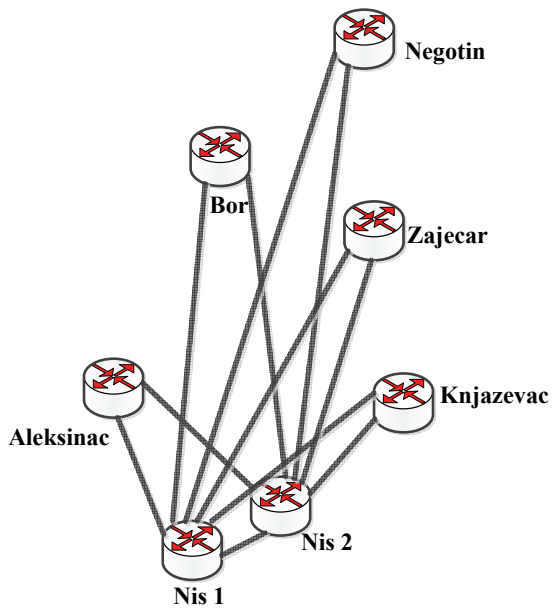**Topology 1**
Two-terminal : 0.9999980
All-terminal : 0.9999951

Fig. 5. Topology 2.

Topology 3
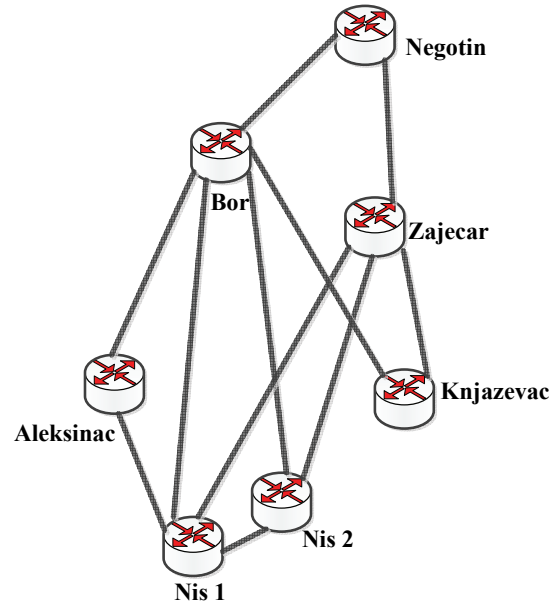*Two-terminal* : 0.9999977
All-terminal : 0.9999964

Fig. 4. Topology 1.

Topology 2
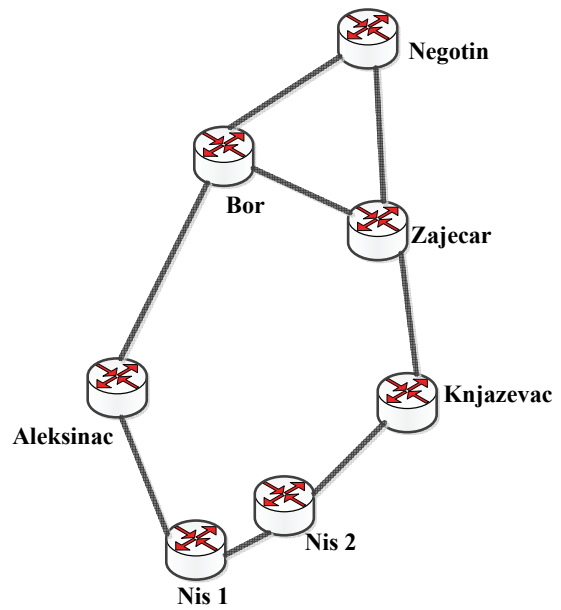*Two-terminal* : 0.99999975
All-terminal : 0.9999983

Fig. 6. Topology 3.

It is now necessary to multiply these results by expression (7), and based on data given in [2], final All-terminal availability values are shown in Table 1.

TABLE 1: FINAL CALCULATED ALL-TERMINAL AVAILABILITY VALUES.

A(t)	Topology 1	Topology 2	Topology3
Redundant nodes	0.9860789	0.9860821	0.986080
Non-redundant nodes	0.9999251	0.9999283	0.999926

It should be noted that these calculated availability values are maximum possible due to the assumption of automatic traffic redirection and other ideal QoS (Quality of Service) parameters also. In real networks, because these values are not ideal, it is obvious that the availability on service level will be even smaller and deviate more from the values obtained in paper [2] (moreover, obtained results with ideal QoS parameters are significantly smaller than those in [2]), which is in favor that availability models in [2] have a large deviation compared to exact availability models presented in this paper. Additionally, it can be observed that cross-link in topology 3 (in [2]), inconsiderably improved the availability, so if only availability is used as a criterion, the best topology is ring, because it gives practically the same availability as the topology 3, but ring topology saves in addition in case of total optical links length in network. Furthermore, there is a correct conclusion in [2] that the "*Telekom Serbia South Network*" topologies 1 and 2 have many more links than they should need in terms of high availability and this attitude is precisely proved by developing the twoall.1.3.jar software and using it for availability calculation in this paper.

IV. CONCLUSION

During the analysis of network topologies given in paper [2], the following conclusions are derived and can be used in other complex network topologies:

1. From equation (5), it is shown that it is not correct to develop only one of the network elements (topology with links only or nodes only). Otherwise, it is necessary to uniformly raise the availability of all network elements.
2. The total availability of the network will certainly be less than availability of only nodes or only network topology.
3. It was shown that the protocols' convergence time and availability of a link do not depend on each other.

In this paper, the basement for broader, further research was set. That future research is going to provide a proposal for an optimal method for designing the network core from the standpoint of high availability and the lowest possible realization price. In future networks which will be quite more complex and larger, it is necessary to investigate will the All-Terminal availability criterion be too sharp and maybe it should be substituted with a less demanding one, to which the authors will continue to pay attention and try to give answers and evidence. Also, based on what it has been seen during the preparation of this paper, it should be insisted on exact models and algorithms with as much as possible greater accuracy in order to reach an optimally designed network topology.

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