Applications of Multifractals in the Analysis of Room Impulse Response - Initial Research

Milan Pavlović and Dragan M. Ristić

Abstract **— The analysis of impulse response has a central place in room acoustics. The research presented in this paper was aimed at examining possibilities for applying multifractal theory in room acoustics focusing on the analysis of room impulse response. Observing an impulse response as a signal with multifractal features gives an opportunity for a new approach in determining the acoustic properties of a sound field in a room. Through the use of characteristic values, gained from multifractal spectra, authors have tried to quantify impulse responses of different rooms. The method used in this paper, together with achieved results, is displaying some of the directions which can be followed when applying multifractals in the field of room acoustics.**

Keywords **— impulse response, multifractals, multifractal spectrum, room acoustic.**

I. Introduction

In every acoustic design of the room, an important role is given to the analysis of its acoustic impulse response, due to the fact that each space gives a unique mark to the structure of its impulse response. Concert and conference halls, churches, audio recording studios, listening rooms, they all have specific acoustics and different impulse responses.

A room impulse response contains all information about acoustics of the room between two specific positions of the source and receiver, [1]. It is used to evaluate a comprehensive set of parameters, most notably the frequency-dependent reverberation time [2]. While the sound is being transmitted from the source to the receiver in room, the existence of reflecting sound energy represents the basic characteristic of the room from the perspective of the acoustic transfer system [2].

A typical look of room impulse response is shown in Fig. 1. and its generalized version (idealistic pattern) in Fig. 2. When a short sound impulse is emitted by the source in the room, the sound wave propagates around in every possible direction, and then gets reflected from the walls and other obstacles in the room. The impulse response of the room emerges at the moment when direct sound reaches a selected receiving point. All of the remaining components visible in the response after direct sound represent various reflections which arrived with a smaller or greater delay regarding to the direct sound.

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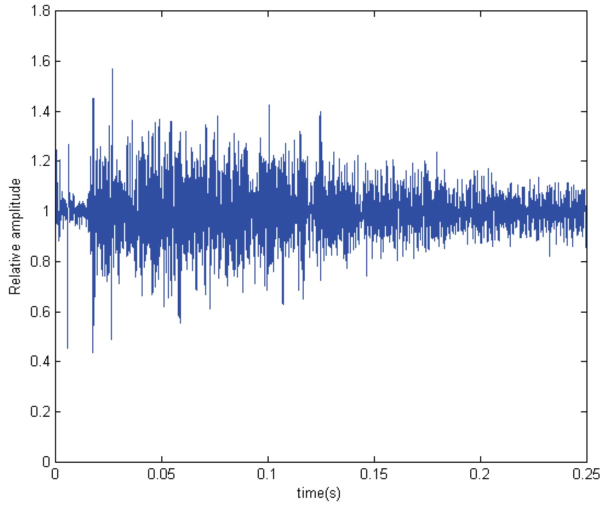

Fig. 1. Example of realistic acoustic impulse response (recorded in the hall of the Opera House in Ljubljana).

Early reflections contribute to the impression of the direct sound in a specific way that is perceived as increased loudness, they support intelligibility of speech, music clarity and the impression of the auditory source width [3]. Such reflections can become a real issue due to their correlations with direct sound.

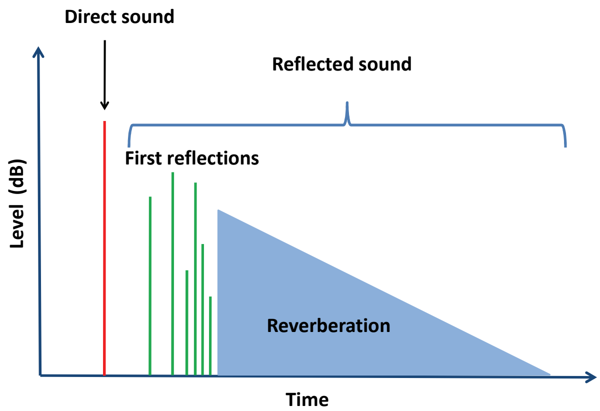

Fig. 2. Generalized version of room impulse response.

After first reflections, follows the reverberation part of the impulse response, as a consequence of forming the subsequent sound reflections in the space, which fill out the impulse response with energy along the time scale. The biggest variations in the form of the impulse response of different rooms appear to be in the period of time in which a response lasts, known as reverberation time.

Important variations are also in the structure of the first reflections, which depends on room dimensions and its acoustic design [2].

The impulse response is carrying an enormous amount of information. Thus, what is apparent from here is the question of finding a manner for reducing such a large amount of information to those parts which are essential for room acoustics. Furthermore, it would also be necessary to explain the fact that our impression of sound in different rooms is different. Answers to these questions are offered by three theories, on which the whole of room acoustics relies: statistics, wave and geometric theory [4].

This paper examines the possibilities of applying multifractals in the field of room impulse response analysis and its quantification through the use of multifractal spectrum. The signal does not have to be a „fractal" in order to be properly analysed with „fractal methods" [5]. The question arising here is not whether the room impulse response is fractal, but rather what kind of local scaling features has a signal, and if there is a simple geometrical or statistical global description of these features using fractal theory. This paper is structured as follows: in the next section we give a short insight into multifractal theory and the basis for application of multifractals in acoustics. Section III lays out a description of the methodology we used, while the results can be found in section IV.

II. Impulse Response As A Signal With Multifractal Features

Multifractals have appeared for the first time in the papers of B.B. Mandelbrot, the creator of fractal geometry [6], who conceived and developed a new geometry of nature and implemented its use in a number of diverse fields [7]. The fractal geometry is based on the idea that forms which seem complex actually exhibit one fundamental feature, known as self–similarity, which can be quantified with an adequate fractal dimension. The study of long-term dynamic behaviour of a physical system can then be attempted by characterization of the fractal properties of a measure that can be associated with the nonuniform distribution. Two different types of multifractality in time series can be distinguished [8]: (*i*) Multifractality due to a broad probability density function for the values of time series, and (*ii*) Multifractality due to different long-range (time-) correlations of small and large fluctuations.

In the last few years, numerous algorithms have been developed with the purpose of segregating characteristic multifractal parameters from a chosen set of data [9]. Due to the nature of room reflections, a room impulse response can be treated as a long-range process. Room impulse responses exhibit self-similarity and hence can be the subject of multifractal analysis.

Generally speaking, during multifractal analysis we follow a certain S structure, which is further divided into non-overlapping boxes S_i with a side ε, whilst $S=\cup_i S_i$. Each box is characterised by some kind of measurement value $\mu(S_i)$. In order to describe multifractals, the so called Hölder exponent is introduced [10]:

$$\alpha_i = \frac{\ln(\mu(S_i))}{\ln(\varepsilon)} \quad (1)$$

wherein the α represents its limit value, when ε→0:

$$\alpha = \lim_{\varepsilon \to 0}(\alpha_i) \quad (2)$$

Parameter α is dependent on its position within the structure and concerns the local regularity. As a measure, $\mu(S_i)$, within the boxes observed during the determination of Hölder exponents, various values can be used: maximum, minimum, sum, deviation, etc. These measures are normalised to a sum of all values in the observed unit.

The global characteristic of observed measure is determined through multifractal spectrum, so-called function $f(\alpha)$, which shows distribution of boxes with Holder's exponent within a range of $\alpha + d\alpha$.. In the case of strictly fractal objects or monofractals, all of the points have the same value of exponents, thus that multifractal spectrum is represented by a single point.

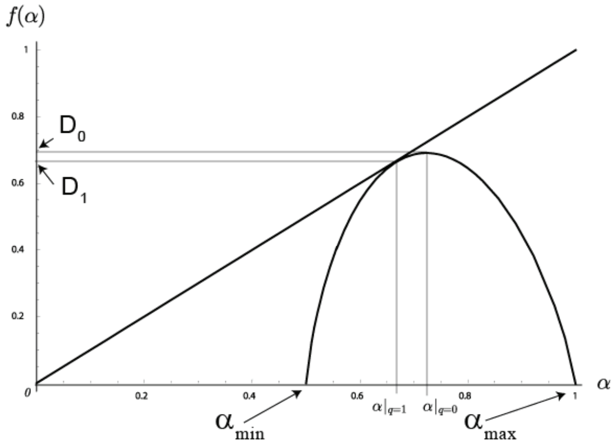

Fig. 3. Example of multifractal spectrum, $f(\alpha)$.

Based on the pair (α, $f(\alpha)$), a signal can be described from both local and global points of view. Small values of α determine a signal which changes poorly on the local scale. On the other hand, small values of $f(\alpha)$ lead to a conclusion that a physical value with a local value α has a small probability of existence, and vice-versa in the case of significantly high values of $f(\alpha)$ [11].

A multifractal spectrum (MF spectrum) represents the ground result of the multifractal analysis (MA). It is a function which quantifies the number of points having the same singularity. Without getting any deeper into the explanation of the term of MF spectrum (shown in Fig. 3.), for the sake of impulse response analysis, some characteristic values of the spectrum have been used: α_{min}, α_{max}, α_0, and $\Delta\alpha$ (width of the spectrum gained as subtraction of α_{max} and α_{min}).

Today there are various methods for determining MF spectrum, i.e. function $f(\alpha)$. One of them is MDFA (*Multifractal detrended fluctuation analysis*) [12], which has been applied in a multitude of different fields and among them in acoustics as well [13], [14]. Evaluation and analysis of the multifractal characteristics of the impulse response have been done using the method of great deviations (*Continuous large deviation*) [15]. MF spectrum calculated by this method offers a compromising solution, lying between precision and complexity of the algorithm.

For the purpose of this paper, MF spectra of the impulse responses are calculated using Fraclab [16]. FracLab is a general purpose signal and image processing toolbox for the Matlab, which enables studying of irregular and at the same time random signals, through the use of the fractal methods [17]. Fraclab is developed by INRIA (*Institut national de recherche en informatique et automatique*). MF spectra have been calculated using continuous large deviation spectrum estimation method in Fraclab, through the use of pure time-domain algorithms [15].

Usage of multifractal theory in the analysis of a room impulse response represents a novelty in room acoustics. In several studies [13], [14], [18], an impulse response has been presented as a signal which has the ability of expressing a similarity to itself. Therefore it can be presented with its characteristic MF spectrum. These papers lead to a conclusion that MF spectrum has certain information regarding the sound field within a room. It has been determined that the width of the multifractal spectrum is related to the complexity of the acoustic impulse response structure [13].

III. Some Of the Possible Approaches to The Analysis

Application of new methods for signal analysis always carries a dilemma of choosing the appropriate way for their implementation. In order to investigate the possibility for applying multifractal analysis to the room impulse response, the global and local properties of signals are considered. Prior to applying multifractal analysis to an impulse response, it is necessary to choose appropriate signals. There are two types of room impulse response signals used in this research. One was recorded in real-world (opera and concert halls, parliament hall, etc.) and the other type were signals recorded in a physical model of room. A physical model of room is an irregular paralellepiped made of 6 glass walls, only two of which are parallel.

In order to make the impulse response characteristics of interest more visible, easier to identify and quantify, it is necessary to choose an appropriate form of impulse response graphical representation for the purpose of analysis. Representation of the room impulse response can be in the form of a bipolar or unipolar signal in time or frequency domain.

The global properties of room impulse response were observed by calculating the MF spectrum of total signal. The analysis of local properties was carried out by calculating the MF spectra of the parts of impulse response covered with a corresponding window. Two methods were used for the analysis of local properties: the expansion (spreading) of the window and shifting (sliding) of the window. A window covers a part of the impulse response signal for which the MF spectrum is calculated. From spectra calculated in such a way typical values were taken and their change was observed.

The expansion of the window method is illustrated in Fig. 4. A red line shows a change in width of the MF spectrum ($\Delta\alpha$) obtained by expanding the window which covers the signal. The graph which shows the change of spectrum width was obtained by calculating the width of the MF spectrum for every step of window expansion (the starting point of the window is fixed, while the width of the window increases at each step by 2 ms). In this way, parts in the impulse response that significantly influence the shape (width) of MF spectrum are found.

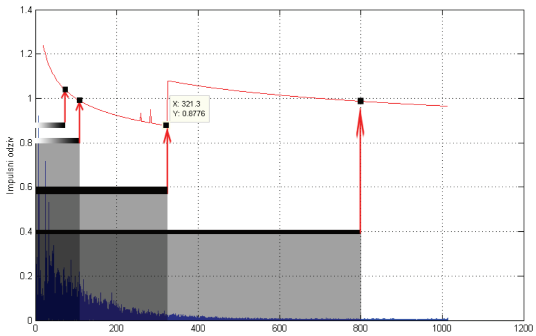

Fig. 4. Illustration of expansion of the window method.

Illustration of the sliding window method is given in Fig. 5. A window of a fixed width (100 ms case shown in Fig. 5.) was shifted in steps of 2 ms. For each part of the impulse response covered by the window the MF spectrum was calculated and characteristic values from the spectrum were extracted, in the case shown in Fig. 5 this value is the width of the spectrum (red line). Unlike the case with the expansion of the window, here the characteristic values of the MF spectrum are recorded at the beginning of the window, so the big jump that in Fig. 4. appears in 321 ms, in the graphics in Fig. 5. was detected in 221 ms, therefore 100 ms earlier, what is the width of the window. For each signal a comparison was made for two different representations of the impulse response (unipolar and bipolar).

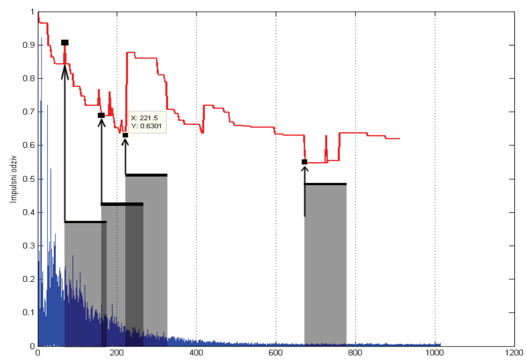

Fig. 5. Illustration of sliding of the window method.

IV. Results

In the following figures some results obtained in our research are presented. Two types of results are shown, one presenting the multifractal spectra of impulse responses used in the research, and the other presenting changes of the characteristic parameters ($\alpha_{\min}$, $\alpha_{\max}$ and α_0) of multifractal spectrum with time.

Multifractal spectra calculated for impulse responses recorded in the physical model of the room are presented in Fig. 6. There were four impulse responses recorded in various configurations with (dotted lines) and without absorbent materials (solid lines) when a microphone was near (red lines) and far away from the source (blue lines). The obtained spectra show broadening of the spectra as walls of the room are covered with absorbent materials.

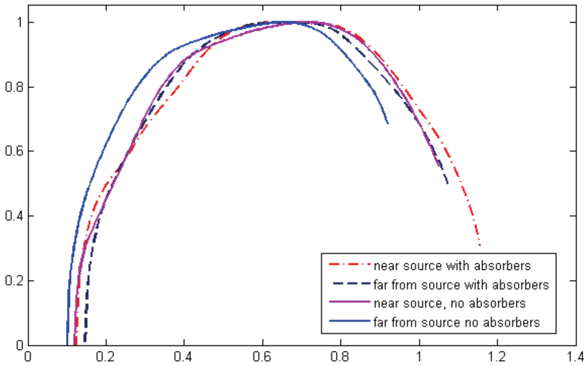

Fig. 6. Multifractal spectra for impulse responses recorded in physical model of room.

The MF spectra calculated for real-world impulse responses are presented in Fig. 7. These impulse responses were mainly taken from large rooms (opera and congress halls). The obtained spectra show some differences between the signals originating from rooms with various acoustic characteristics, when the shape of the spectrum and parameter $\alpha_{\min}$ is considered. On the left side of the graph, around the value of $\alpha_{\min}$=0.1, there are spectra of impulse responses from Ljubljana's Opera, Sava Center and *old1* (Maribor opera before the reconstruction) signal, while on the right side above the value of $\alpha_{\min}$=0.3 are signals from the court of the Slovenian parliament. The difference in shape of the spectrum can be seen when impulse responses of Maribor opera hall before and after reconstruction, named *old1* and *new1* (red and violet colored line), are compared. In this example it can be seen clearly how the intervention in acoustical properties of the room influence the change in shape of the multifractal spectrum.

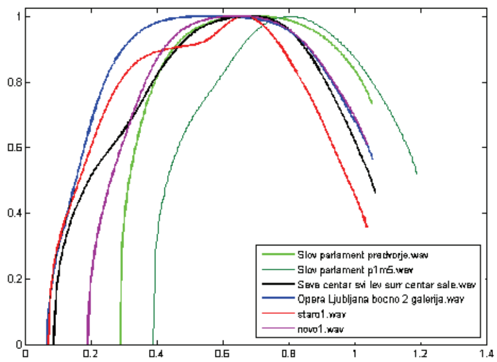

Fig. 7. Multifractal spectra for different real-world impulse responses.

Fig. 8. presents the results obtained by expansion of the window method. Results of the analysis of impulse responses by sliding of the window method (Fig. 8.) are given in three cases for three different values of window width (60, 100 and 200 ms); the step was always 2 ms.

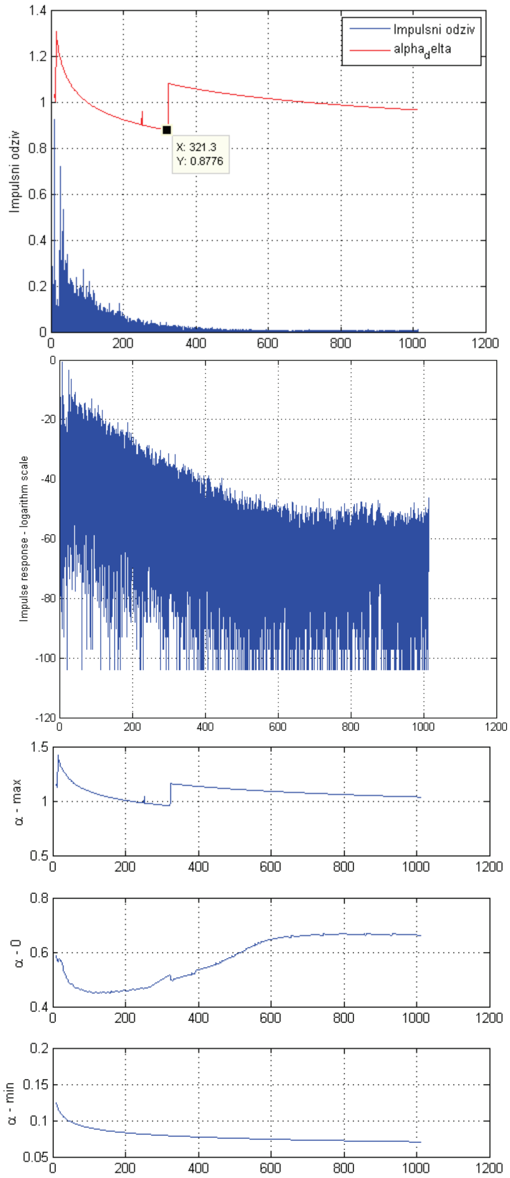

Fig. 8. Change of $\Delta\alpha$, $\alpha_{\min}$, $\alpha_{\max}$ and α_0 parameters obtained with expansion of the window method.

The change of parameter α_{min} shown in Fig. 8. does not provide any significant information, while the change of α_0 shows a certain degree of correlation with the observed signal of room impulse response in a logarithmic scale. Graphs which change of α_{max} and width of the spectrum (red line in Fig. 6.) indicate the existence of "anomalies" in the signal that lead to a rapid increase in the width of multifractal spectrum. The existence of some "anomalies" in the signal of the impulse response that cause jumps in the parameters of the MF spectrum is also evident in the panels in Fig. 9.

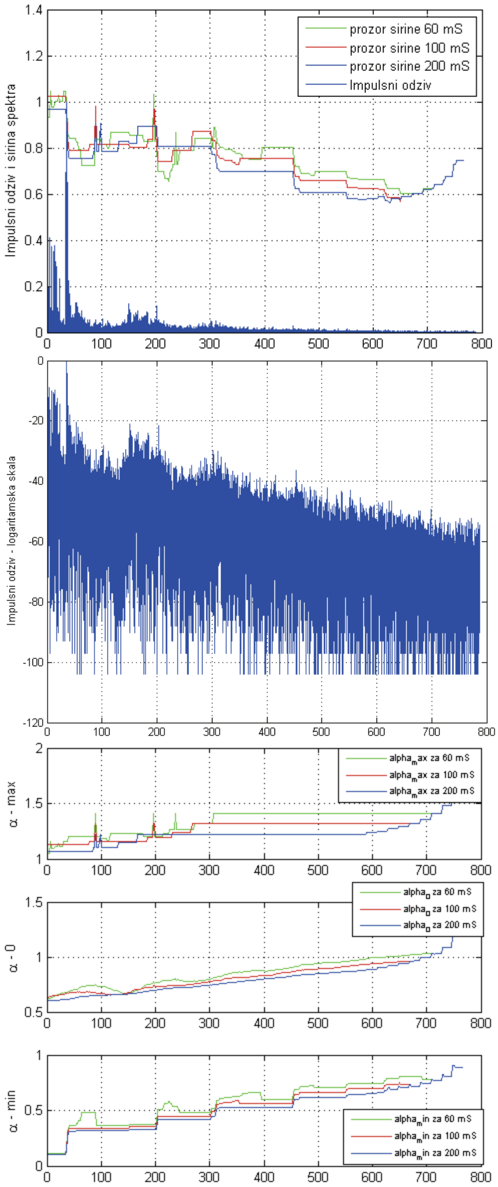

Fig. 9. Change of $\Delta\alpha$, $\alpha_{\min}$, $\alpha_{\max}$ and α_0 parameters obtained with sliding window method for three different values of window width (60, 100 and 200 ms).

V. Conclusion

An impulse response is of great importance in room acoustics since it is the "identity card" of the room and hence its analysis takes a central place in room acoustics. Investigating room impulse response as a signal with multifractal properties provides new opportunities to understand this signal. That is exactly what this paper tried to show by presenting some of the possible ways to implement multifractal analysis on the room impulse response. Many questions remain open for discussion, which will be the subject of future work. Some of them are: how does a multifractal spectrum quantify the acoustic properties of the room and what does it tell us about the established sound field in the room? Which phenomena in the impulse response can be observed with the MF spectrum? What does a multifractal spectrum actually show us and what can we measure with it in room acoustics? The inverse problem is also a big challenge, i.e. how to extract specific parts of the impulse response from a multifractal spectrum.

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