OFDM AF Variable Gain Relay System for the Next Generation Mobile Cellular Networks

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Abstract — **In this paper we present analytical performance evaluation of a dual-hop OFDM amplify-and-forward (AF) variable gain (VG) relay system implementing ordered subcarrier mapping (SCM) at the relay station (R), considered to be a very interesting solution for the next generation mobile cellular networks. A scenario with no direct communication between the source of information (S) and destination terminal (D), with the Rayleigh fading statistics on both hops is assumed. A closed form analytical expression for the bit error rate (BER) performance of the considered system with DPSK modulation is derived, while for its ergodic capacity performance, a tight upper bound expression is obtained. The accuracy of the undertaken analytical approach is confirmed through comparison with simulation results. It is shown that significant capacity enhancement can be achieved through SCM implementation at R, for all the signal-to-noise ratio (SNR) values on both hops, but especially in the region of small SNRs on hops. BER analysis reveals that in the region of small and medium average SNRs on both hops BER performance may also be improved with SCM at R station.**

Key words — **BER, ergodic capacity, OFDM AF relay system, subcarrier mapping, variable gain.**

I. INTRODUCTION

RELAY systems have become a subject of intensive research interest, as it is recognized that they can improve capacity and extend the coverage area of wireless communication systems. Additional attention is focused on them since they were proposed for the implementation in mobile cellular networks of the new generation, [1]. Many research efforts conducted from that period have led to the acceptance of two standards that are recognized as IMT-Advanced, or 4G systems, [2]. Both of them assume implementation of relay (R) stations in dual-hop scenarios with three communication terminals, where there is no direct communication between the source of information (S) and destination terminal (D). The R station performs signal processing based on a decode-and-forward (DF) paradigm, which means that the signal received from S is fully decoded, and then again re-encoded before forwarding towards D. However, it is expected that some of the upcoming IMT-Advanced systems specifications will involve amplify-and-forward (AF) relaying, where the R station only amplifies the signal received from R, and then forwards it to D, [2], [3]. In such a scenario the R station may amplify the received signal with a fixed gain (FG), or with a variable gain (VG), depending on its ability to estimate S-R channel. AF relay systems are simpler for realization and introduce shorter latency than the DF relay systems. Thus, in this paper we analyze the performance of relay systems implementing AF VG relaying.

Like most of the modern broadband wireless communication systems, the next generation cellular networks will employ orthogonal frequency division multiplexing (OFDM) on a physical layer, due to its proven ability to provide high data rates even in frequency selective channels, [3]. Nowadays, numerous researches are ongoing with the aim to define new solutions for performance improvement of the standardized OFDM relay systems for cellular networks. One of the interesting solutions for performance improvement is the implementation of subcarrier mapping (SCM) at the R station, which can enhance the system capacity, [4]-[8], and/or improve bit error rate (BER) performance, [7]-[10]. For the capacity maximization, the so called best-to-best SCM (BTB SCM) scheme should be used, where the subcarrier with the highest SNR from the first hop should be mapped to the subcarrier with the highest SNR on the second hop, then the subcarrier with the second highest SNR from the first hop to the subcarrier with the second highest SNR on the second hop, etc, [4]-[8]. However, for dual-hop OFDM AF relay systems, it has been shown in [9], that BTB SCM scheme improves the BER performance only in the region of small and medium SNRs on both hops, while for high SNRs, the SCM scheme denoted as BTW SCM should be implemented. This SCM scheme assumes that the subcarrier with the highest SNR from the first hop should be mapped to the subcarrier with the lowest SNR on the second hop, etc. Ergodic capacity performance of OFDM AF FG relay system with SCM has been analyzed through different approaches in [6] and [7], while its BER performance has been evaluated in [7] and [10]. However, the level of capacity enhancement and BER performance improvement achieved through implementation of SCM at the R station in OFDM AF VG relay system has not been examined yet. Thus, in this paper we perform analytical and simulation evaluation of the performance benefits attained with this new solution for the OFDM based relay systems.

The paper is organized as follows: Section II describes the analyzed OFDM AF VG relay system with SCM and the considered scenario. The analytical derivation of the probability density function (PDF) and moment generation

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function (MGF) of the SNR at D is given in Section III. This Section also contains an analytical approach for derivation of the BER expression for DPSK (Differentially Phase Shift Keying) modulated considered system, as well as derivation of the upper bound of the ergodic capacity. Section IV presents the analytically and simulation obtained BER and ergodic capacity results. Section V concludes the paper.

II. SYSTEM MODEL

We consider an OFDM dual-hop relay system with a source terminal S, a half-duplex relay terminal R and a destination one D, each equipped with a single antenna. A scenario without the possibility of direct communication between S and D is assumed. Orthogonality of the S-R and R-D channels is achieved by dividing a communication process into two time slots. The R terminal has FFT (Fast Fourier Transformation) and IFFT (Inverse Fast Fourier Transformation) blocks for OFDM demodulation and OFDM modulation, respectively. It is assumed that R has perfect channel knowledge of both S-R and R-D channels. Using that, R performs variable gain relaying, where the signal that reaches the relay on the i-th subcarrier is amplified by a gain $G_i=1/H_{1,i}$, with $H_{1,i}$ being the i-th subcarrier channel transfer function. Furthermore, R has a block that performs subcarrier mapping (SCM), mapping subcarriers from the first hop to subcarriers on the second hop in accordance with their instantaneous SNRs. In order to perform signal demodulation it is necessary that D knows the permutation function performed at R. Fig. 1 presents the simplified block scheme of the relay station in OFDM variable gain relay system with SCM.

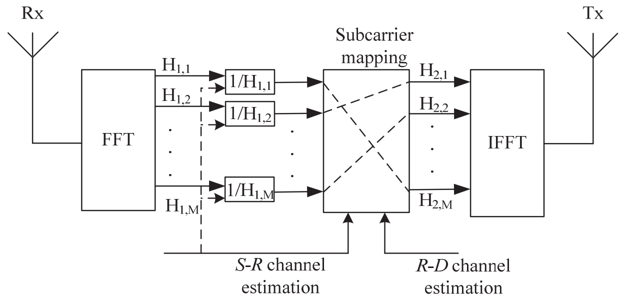

Fig. 1. OFDM AF VG relay station with SCM.

The post-FFT signal on the i-th subcarrier, received at the relay station, is given by:

$$Y_{R,i} = X_i H_{1,i} + N_{1,i}, \quad 1 \le i \le M \tag{1}$$

where M is the total number of subcarriers and X_i is a data symbol sent by source on the i-th subcarrier. $N_{1,i}$ represents an additive white Gausssian noise on the i-th subcarrier with variance $\mathbf{E}(|N_{1,i}|^2)=N_{01}$, $\mathbf{E}(\cdot)$ denoting the expectation operator. Assuming that the SCM function $v(i)$ performs mapping of the i-th subcarrier from the first hop to the k-th subcarrier on the second hop, the frequency domain signal at D can be written as:

$$\begin{aligned} Y_{D,k} &= G_i H_{2,k} Y_{R,v(i)} + N_{2,k} \\ &= G_i H_{2,k} H_{1,i} X_i + G_i H_{2,k} N_{1,i} + N_{2,k}, \quad 1 \le k \le M, \end{aligned} \tag{2}$$

with $H_{2,k}$ denoting the k-th subcarrier channel transfer function on the second hop. $N_{2,k}$ is an additive white Gaussian noise on the k-th subcarrier at the destination, having variance $\mathbf{E}(|N_{2,k}|^2)=N_{02}$. Fadings in the S-R and R-D channels are assumed to be independent and identically distributed (i.i.d.) with the Rayleigh distribution, resulting in the exponential form of PDF of SNR on both channels. Using (2) and the described gain G_i applied at the subcarrier level at R, the instantaneous SNR on the k-th subcarrier at D can be presented as:

$$\gamma_{k,end} = \frac{\gamma_{i,SR}\gamma_{k,RD}}{\gamma_{i,SR} + \gamma_{k,RD}}, \tag{3}$$

where $\gamma_{i,SR}$ and $\gamma_{k,RD}$ denote instantaneous SNR on the i-th subcarrier of the S-R link and on the k-th subcarrier of the R-D link, respectively.

III. PERFORMANCE ANALYSIS

In order to attain performance analysis of OFDM AF VG relay system implementing SCM, it is necessary to know PDF of the SNR per subcarrier at D.

A. PDF of SNR

The PDF of SNR for the k-th "weakest" subcarrier, meaning that it has the k-th lowest SNR out of the M total ones on the S-R link, for the assumed Rayleigh fading, is given in [7] as:

$$f_k^w(x) = \sum_{i=0}^{k-1} \lambda_{SR} \alpha_i e^{-\beta_i \lambda_{SR} x}, \tag{4}$$

where $\lambda_{SR} = 1/\overline{\gamma}_{SR}$, and $\overline{\gamma}_{SR}$ denotes the average SNR on the S-R link. The coefficients α_i and β_i have the values:

$$\alpha_i = (-1)^i M \binom{M-1}{k-1} \binom{k-1}{i}, \; \beta_i = i + M - k + 1, \tag{5}$$

with (:) representing the binomial coefficient.

The PDF of SNR for the k-th "strongest" subcarrier on the R-D link, meaning that it has the k-th highest SNR out of the M total ones, can be written as in [7]:

$$f_k^s(x) = \sum_{i=0}^{M-k} \lambda_{RD} \alpha_i e^{-\beta_i \lambda_{RD} x}. \tag{6}$$

Here, $\lambda_{RD} = 1/\overline{\gamma}_{RD}$ with $\overline{\gamma}_{RD}$ representing the average SNR on the R-D link. The introduced coefficients are equal to:

$$\delta_i = (-1)^i M \binom{M-1}{k-1} \binom{M-k}{i}; \; \varepsilon_i = i + k. \tag{7}$$

Assuming that the random variable $\gamma_{k,\mathrm{SR}}$ denotes the instantaneous SNR of the k-th weakest subcarrier on the S-R link, and $\gamma_{k,\mathrm{RD}}$ the instantaneous SNR of the k-th strongest subcarrier on the R-D link, the harmonic mean of these two random variables can be written as

$$\mu_H(\gamma_{k,SR}, \gamma_{k,RD}) = \frac{2}{\dfrac{1}{\gamma_{k,SR}} + \dfrac{1}{\gamma_{k,RD}}} = \frac{2\gamma_{k,SR}\gamma_{k,RD}}{\gamma_{k,SR} + \gamma_{k,RD}} = 2\gamma_{k,end}. \tag{8}$$

Using (4) and (6), and following the same steps as in [11] for deriving the PDF of the harmonic mean of two exponentially distributed random variables, we derived the PDF of $\mu_H(\gamma_{k,\mathrm{SR}}, \gamma_{k,\mathrm{RD}})$, and through this the PDF of SNR for the k-th subcarrier at D in the case of BTW SCM:

$$f_{\gamma_k,end}^{BTW}(x) = 2\sum_{j=0}^{k-1}\sum_{i=0}^{M-k}\frac{\alpha_j\delta_i}{\overline{\gamma}_{SR}\overline{\gamma}_{RD}}xe^{-x\left(\frac{\beta_j}{\overline{\gamma}_{SR}}+\frac{\varepsilon_i}{\overline{\gamma}_{RD}}\right)}\left[\frac{\overline{\gamma}_{RD}\beta_j+\overline{\gamma}_{SR}\varepsilon_i}{\sqrt{\overline{\gamma}_{SR}\overline{\gamma}_{RD}\varepsilon_i\beta_j}}\right. \quad (9)$$
$$\left.\times K_1\left(2x\sqrt{\frac{\beta_j\varepsilon_i}{\overline{\gamma}_{SR}\overline{\gamma}_{RD}}}\right)+2K_0\left(2x\sqrt{\frac{\beta_j\varepsilon_i}{\overline{\gamma}_{SR}\overline{\gamma}_{RD}}}\right)\right],$$

where $K_0(\cdot)$ and $K_1(\cdot)$ are zero and first order modified Bessel functions of the second kind defined in [12, eqs. (9.6.21), (9.6.22)].

The PDF of SNR for the k-th subcarrier at D in the case of BTB SCM scheme implemented can be obtained following the same derivation steps as the ones for the BTW SCM, but assuming that subcarriers in both hops are increasingly ordered according to their instantaneous SNRs. The final expression is then derived in the form:

$$f_{\gamma_k,end}^{BTB}(x) = 2\sum_{j=0}^{k-1}\sum_{i=0}^{k-1}\frac{\alpha_j\alpha_i}{\overline{\gamma}_{SR}\overline{\gamma}_{RD}}xe^{-x\left(\frac{\beta_j}{\overline{\gamma}_{SR}}+\frac{\beta_i}{\overline{\gamma}_{RD}}\right)}\left[\frac{\overline{\gamma}_{RD}\beta_j+\overline{\gamma}_{SR}\beta_i}{\sqrt{\overline{\gamma}_{SR}\overline{\gamma}_{RD}\beta_i\beta_j}}\right. \quad (10)$$
$$\left.\times K_1\left(2x\sqrt{\frac{\beta_j\beta_i}{\overline{\gamma}_{SR}\overline{\gamma}_{RD}}}\right)+2K_0\left(2x\sqrt{\frac{\beta_j\beta_i}{\overline{\gamma}_{SR}\overline{\gamma}_{RD}}}\right)\right].$$

B. *BER Performance Analysis*

We used the moment generating function (MGF) based approach for the BER performance analysis of the DPSK modulated OFDM AF VG relay system employing SCM at R station. Using the known PDF of SNR, the MGF of SNR for the k-th subcarrier at D in the considered relay system is derived using [13, eq. (6.621.3)]:

$$\mathcal{M}_{\gamma_{k,end}}(s) = \mathbf{E}\left(e^{-\gamma s}\right) = \frac{16}{3}\sum_{j=0}^{k-1}\sum_{i=0}^{M-k}\frac{\alpha_j\delta_i}{\overline{\gamma}_{SR}\overline{\gamma}_{RD}}\frac{1}{\left(s+L_{j,i}+2\sqrt{B_{j,i}}\right)^2}$$
$$\times\left[\frac{4L_{j,i}}{s+L_{j,i}+2\sqrt{B_{j,i}}}\,{}_2F_1\left(3,\frac{3}{2};\frac{5}{2};\frac{s+L_{j,i}-2\sqrt{B_{j,i}}}{s+L_{j,i}+2\sqrt{B_{j,i}}}\right)\right. \quad (11)$$
$$\left.+\,{}_2F_1\left(3,\frac{3}{2};\frac{5}{2};\frac{s+L_{j,i}-2\sqrt{B_{j,i}}}{s+L_{j,i}+2\sqrt{B_{j,i}}}\right)\right].$$

In (11), ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gaussian hypergeometric function defined in [12, eq. (9.100)], while the introduced coefficients are equal to $L_{j,i} = \beta_j/\overline{\gamma}_{SR} + \varepsilon_i/\overline{\gamma}_{RD}$ and $B_{j,i} = \beta_j\varepsilon_i/\overline{\gamma}_{SR}\overline{\gamma}_{RD}$.

For the OFDM AF VG relay system implementing BTB SCM, the MGF of SNR for the k-th subcarrier at D is obtained in the form:

$$\mathcal{M}_{\gamma_{k,end}}(s) = \frac{16}{3}\sum_{j=0}^{k-1}\sum_{i=0}^{k-1}\frac{\alpha_j\alpha_i}{\overline{\gamma}_{SR}\overline{\gamma}_{RD}}\frac{1}{\left(s+I_{j,i}+2\sqrt{A_{j,i}}\right)^2}$$
$$\times\left[\frac{4I_{j,i}}{s+I_{j,i}+2\sqrt{A_{j,i}}}\,{}_2F_1\left(3,\frac{3}{2};\frac{5}{2};\frac{s+I_{j,i}-2\sqrt{A_{j,i}}}{s+I_{j,i}+2\sqrt{A_{j,i}}}\right)\right. \quad (12)$$
$$\left.+\,{}_2F_1\left(3,\frac{3}{2};\frac{5}{2};\frac{s+I_{j,i}-2\sqrt{A_{j,i}}}{s+I_{j,i}+2\sqrt{A_{j,i}}}\right)\right]$$

with $I_{j,i} = \beta_j/\overline{\gamma}_{SR} + \beta_i/\overline{\gamma}_{RD}$ and $A_{j,i} = \beta_j\beta_i/\overline{\gamma}_{SR}\overline{\gamma}_{RD}$.

Having the closed form solution for the MGF of SNR for the k-th subcarrier at D in both BTB and BTW SCM schemes, the average BER for the DPSK modulated OFDM AF relay system with the SCM can be evaluated as:

$$P_b = \frac{1}{M}\sum_{k=1}^{M}\frac{1}{2}\mathcal{M}_{\gamma_{k,end}}(1)\cdot \quad (13)$$

C. *Ergodic Capacity Analysis*

As it is proven that the BTB SCM scheme achieves the highest capacity among all the other possible SCM schemes, in this part we analyze the ergodic capacity bound of the OFDM AF VG relay system employing BTB SCM. The ergodic capacity normalized to a unit bandwidth for the k-th subcarrier at D, in the considered relay system can be calculated using the obtained PDF of SNR for the k-th subcarrier at D:

$$C_k = \frac{1}{2}\mathbf{E}\left(\log_2(1+\gamma_k)\right) = \frac{1}{2}\int_0^{\infty}\log_2(1+\gamma)f_{\gamma_{k,end}}^{BTB}(\gamma)d\gamma\cdot \quad (14)$$

The factor 1/2 comes due to transmission over two time-slots. Having $f_{\gamma_{k,end}}^{BTB}(\gamma)$ as in (10) precludes finding a closed-form solution for the integral in (14). However, the ergodic capacity can be upper bounded using Jensen's inequality [7] and noting that log(·) is a concave function, yielding:

$$C_k \le \frac{1}{2}\log_2(1+\mathbf{E}(\gamma_k))\cdot \quad (15)$$

The expectation of the k-th subcarrier SNR at D is determined as:

$$\mathbf{E}(\gamma_k) = \int_0^{\infty}\gamma f_{\gamma_{k,end}}^{BTB}(\gamma)d\gamma = \frac{2}{\overline{\gamma}_{SR}\overline{\gamma}_{RD}}\sum_{j=0}^{k-1}\sum_{i=0}^{k-1}\alpha_j\alpha_i\left(\mathcal{I}_1+\mathcal{I}_2\right), \quad (16)$$

where $\mathcal{I}_1$ and $\mathcal{I}_2$ denote the integrals:

$$\mathcal{I}_1 = \frac{I_{j,i}}{\sqrt{A_{j,i}}}\int_0^{\infty}\gamma^2 e^{-\gamma I_{j,i}}K_1\left(2\gamma\sqrt{A_{j,i}}\right)d\gamma, \quad (17)$$

$$\mathcal{I}_2 = 2\int_0^{\infty}\gamma^2 e^{-0.5\gamma I_{j,i}}K_0\left(2\gamma\sqrt{A_{j,i}}\right)d\gamma\cdot \quad (18)$$

Using the integral solution given in [13, eq. (6.621.3)], the closed-form expressions for $\mathcal{I}_1$ and $\mathcal{I}_2$ in (17) and (18), respectively, are found, thus obtaining:

$$\mathbf{E}(\gamma_k) = \frac{128}{15\overline{\gamma}_{SR}\overline{\gamma}_{RD}}\sum_{j=0}^{k-1}\sum_{i=0}^{k-1}\frac{\alpha_j\alpha_i}{\left(I_{j,i}+2\sqrt{A_{j,i}}\right)^3}\left[\frac{3I_{j,i}}{I_{j,i}+2\sqrt{A_{j,i}}}\right. \quad (19)$$
$$\left.{}_2F_1\left(4,\frac{3}{2};\frac{7}{2};\frac{I_{j,i}-2\sqrt{A_{j,i}}}{I_{j,i}+2\sqrt{A_{j,i}}}\right)+{}_2F_1\left(3,\frac{1}{2};\frac{7}{2};\frac{I_{j,i}-2\sqrt{A_{j,i}}}{I_{j,i}+2\sqrt{A_{j,i}}}\right)\right].$$

Substituting (19) into (15), an upper bound for the k-th subcarier ergodic capacity in OFDM AF VG relay system with BTB SCM, is derived. Average ergodic capacity per subcarrier is then obtained through averaging over all M subcarriers of the considered system:

$$C = \frac{1}{M}\sum_{k=1}^{M}C_k\,\cdot \quad (20)$$

IV. RESULTS

The subsequent analytical and simulation results assume a perfectly synchronized OFDM AF VG relay system with the implemented SCM at R station. The OFDM system has M=32 subcarriers, which in a real scenario can be considered as 32 chunks with uncorrelated transfer functions from chunk to chunk, [8]. It is also assumed that noise variances at R and D are the same, $N_{01}=N_{02}$. We used Monte Carlo simulations of the considered OFDM relay system, where we have modeled its part that belongs to frequency domain. This can be considered as an adequate approach as we have assumed perfect time and frequency synchronization among S, R and D. In simulations, independent and identically distributed subcarriers were generated as complex random Gaussian variables having zero mean value and variance 1/2, thus modeling the Rayleigh fading distribution where each subcarrier has its power normalized to one.

Fig. 2 gives the analytically obtained, as well as simulated BER performances of the considered OFDM AF VG relay system with SCM. In order to get an insight into the level of BER performance improvement achieved through the implementation of SCM at R station, the BER of the OFDM AF VG relay system with no SCM is presented. A scenario with the equal average SNRs on both hops is assumed.

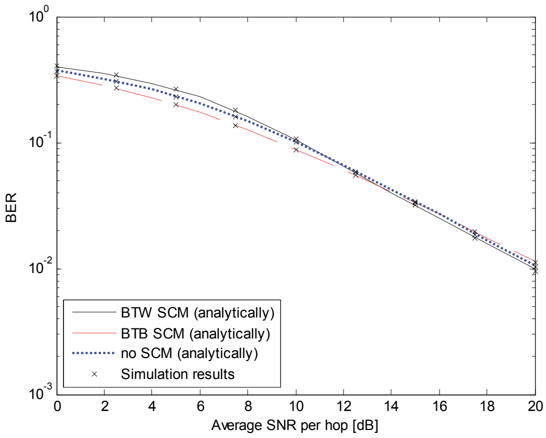

Fig. 2. BER performance of DPSK modulated OFDM AF VG relay system with SCM.

The BER values given in Fig. 2 clearly show that all the analytically obtained results are completely verified by simulations. In other words, it can be concluded that the algorithms introduced for the end-to-end SNR statistics (PDF and MGF) determination can be considered as valid. Further on, it is obvious that the SCM implementation does not offer significant BER performance enhancements for this type of relay systems, when compared with the improvements achieved in the OFDM AF FG relay systems, [7]. Additionally, for the relay system with a variable gain, the best BER values are attained with the BTB SCM scheme when average SNR per hop is below 13dB, while above this SNR value the advantage is on the side of the BTW SCM. However, the level of BER performance improvement achieved through BTW SCM at high average SNR values per hop is very small compared to the system with no SCM. Thus, from the point of view of the achieved BER performance improvement, only the implementation of the BTB SCM scheme can justify the increased system complexity. The system with BTB SCM has the best BER performance in the range of low average SNR values, i.e. when the channel propagation conditions can be considered as unfavorable for the deployment of the system with no SCM. For example, the SNR gain achieved with BTB SCM is equal to 1dB for the BER value of 10^{-1}, when compared with the system with no SCM. Moreover, the BTB SCM scheme is known as the one that maximizes the achievable capacity.

The analytically obtained ergodic capacity values are also compared with the simulation results, for the sake of verification. Simulations include generation of independent Rayleigh fading channel transfer functions, for each subcarrier on the S-R link ($H_{1,k}^{(n)}$) and on the R-D link ($H_{2,k}^{(n)}$), and for each channel realization, where n denotes the n-th channel realization ($1\leq n\leq n_{tot}$). The value of the ergodic capacity for the k-th subcarrier at the system receiving end, for the considered n-th simulation realization is defined as:

$$C_k^{(n)}=\frac{1}{2}\log_2\left(1+\frac{\left(G^{(n)}\right)^2\left|H_{1,k}^{(n)}H_{2,k}^{(n)}X_k^{(n)}\right|^2}{\left(G^{(n)}\right)^2\left|H_{2,k}^{(n)}\right|^2N_{01}+N_{02}}\right). \tag{21}$$

$G^{(n)}$ represents the gain factor introduced at the R station, which corresponds to the n-th S-R link channel realization. The average ergodic capacity for the k-th subcarrier at the receiving end of the considered relay system is then found by averaging over n_{tot} simulation repetitions:

$$C_k=\frac{1}{n_{tot}}\sum_{n=1}^{n_{tot}}C_k^{(n)}, \tag{22}$$

while the average ergodic capacity per subcarrier is then obtained through averaging over all M subcarriers.

Plots given in Fig. 3 represent the analytical and simulation results for the average capacity per subcarrier of the considered OFDM AF VG relay system with BTB SCM, in the scenario assuming equal average SNR on both hops. In order to gain an insight into the level of

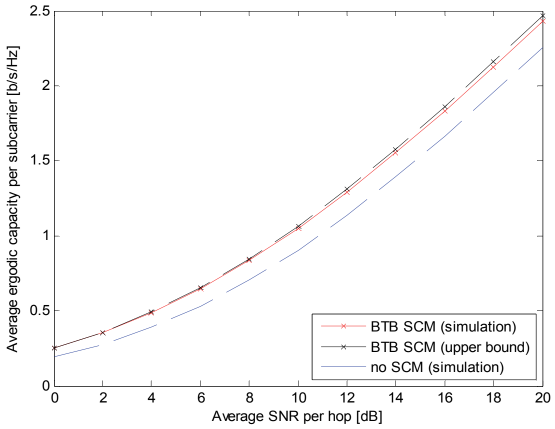

Fig. 3. Ergodic capacity per subcarrier of OFDM AF VG relay system with and without SCM.

capacity enhancement achieved through the BTB SCM implementation at R, the average ergodic capacity for the OFDM AF VG relay system without SCM obtained through simulations is also presented.

From the plots given in Fig. 3 it can be seen that a very tight upper bound of the ergodic capacity has been derived, which confirms the accuracy of the undertaken analytical approach. Namely, the obtained analytical results differ less than 1.5% from the simulation results for all analyzed SNR values on both hops (Table 1), thus approving that they can be used for the ergodic capacity analyses. The comparative analysis of the average ergodic capacity of OFDM AF VG relay system implementing BTB SCM with the ergodic capacity of the system without SCM given in Table 1 shows the level of the capacity enhancement achieved through SCM at R station. It is very important that the greatest capacity enhancement is attained in the region of small average SNRs on both hops, i.e. when the channel conditions are bad and it may happen that the system with no SCM cannot meet the required quality of service. Thus, for example, the system with BTB SCM has 24.8% higher capacity than the system without SCM for the average SNR per hop value of 4dB, while the capacity enhancement achieved through BTB SCM is equal to 7.8% for the average SNR per hop value of 20dB.

TABLE 1: ERGODIC CAPACITY OF OFDM AF VG SYSTEM

Average SNR per hop [dB]	**4**	**10**	**20**
no SCM [b/s/Hz] (simul)	0.387	0.9	2.254
BTB SCM [b/s/Hz] (simul.)	0.483	1.05	2.431
BTB SCM [b/s/Hz] (up. bou.)	0.490	1.065	2.464
Capacity enhancement	24.8%	16.6%	7.8%

Since in mobile cellular systems the R station will be placed in such a position that channel variations between the base station (representing S in the downlink communication process) and R station will be small, then it is interesting to analyze the ergodic capacity performance as a function of the average SNR on the R-D link. The plots given in Fig. 4 assume this kind of scenario, and they confirm the previously conducted conclusions about the significance of the implementation of BTB SCM for the capacity enhancement, especially in the region of small SNRs on both hops. The given plots show that for $\overline{\gamma}_{SR} = 5\text{dB}$ and $\overline{\gamma}_{RD} = 0\text{dB}$ the relay system with BTB SCM achieves 23% higher capacity than the system without SCM, while for the case when $\overline{\gamma}_{SR} = 15\text{dB}$ and $\overline{\gamma}_{RD} = 0\text{dB}$ the capacity enhancement is equal to 8%.

V. CONCLUSIONS

BER and ergodic capacity performance evaluation of the OFDM AF VG relay system implementing SCM at the R station has been conducted. Closed form BER expressions are derived for the DPSK modulated considered system, implementing BTB SCM and BTW SCM. The obtained results are completely verified by simulations. They have shown that, in order to improve BER performances, the BTB SCM scheme should be implemented in the region of small and medium average SNRs on both hops, while for the higher average SNRs SCM should not be implemented.

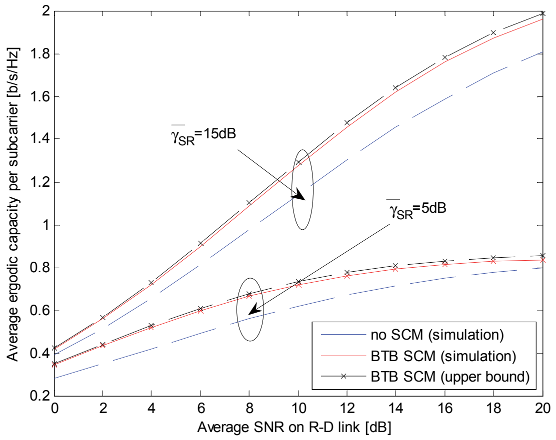

Fig. 4. Ergodic capacity per subcarrier of OFDM AF VG relay system with and without SCM.

Ergodic capacity analysis of the considered system with BTB SCM has resulted in derivation of the very tight upper bound of achieved ergodic capacity, which differs less than 1.5% from the simulation results for all SNR values on both hops. Comparative analysis has shown that very significant capacity enhancement can be achieved through the implementation of BTB SCM at the R station, especially in the region of small SNRs on both hops (up to 30%). This is particularly important as it means that the BTB SCM scheme enables the highest BER performance improvement and capacity enhancement in the worst case scenario. The obtained results have confirmed that the considered OFDM AF VG relay system implementing SCM at the R station can be considered as an interesting solution for the next generation mobile cellular networks.

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