# BER Analysis of STBC Codes for MIMO Rayleigh Flat Fading Channels

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*Abstract* — The paper presents the analysis of Space-Time Block Codes (STBC) used in Multiple-Input Multiple-Output (MIMO) systems to assure transmit diversity. The receive diversity is resolved with a Maximum Ratio Combining technique. For a fixed number of transmit antennas, the performances of different STBC codes are analyzed in terms of Bit Error Rate (BER) and diversity gain for a quasi-static Rayleigh flat fading channel. Finally, it is shown that by increasing the number of transmit/receive antennas, the system performances increase.

*Keywords* — Bit error rate, maximum ratio combiner, receive diversity, space-time block codes, transmit diversity.

#### I. INTRODUCTION

IN modern wireless communication systems like 4G, WLAN and WiMAX, the multipath propagation channel is modeled as a Multiple-Input Multiple-Output (MIMO) system. In order to combat the effects of multipath fading and to increase the capacity and reliability of the wireless channel, a practical solution is the spatial diversity, using multiple antennas at one or both sides of the link [1]-[3].

For MIMO systems, the knowledge of the Channel State Information (CSI) at the transmitter (CSIT) and at the receiver (CSIR) is a primordial criterion for choosing a diversity technique. At the receiver, if the channel is unknown it can be estimated using different techniques so we suppose that we always have CSIR. At the transmitter, if the channel is known (with CSIT) then beamforming techniques are used to assure both the diversity gain and the array gain; if the channel is not known (without CSIT) then Space-Time (ST) codes are used to assure only the diversity gain. The ST codes are a more general class of error correcting codes, with a spatial-temporal structure, the control symbols being inserted in both spatial and temporal domains.

In [4], Foschini introduces the multi-layered space-time architecture, known as Bell Labs Layered Space-Time (BLAST). Later, in [5] are proposed the Space-Time Trellis Codes (STTrC) which provide the best tradeoff between constellation size, data rate, diversity gain and trellis complexity, but with a greater decoding complexity. Addressing the last issue, Alamouti introduced in [6] a simple diversity scheme for two transmit antennas, which provides a maximum diversity gain and no coding gain for a

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This paper is structured as follows. Section II presents a mathematical model for a general MIMO system, particularized for the Alamouti coding at the transmitter and Maximum Ratio Combining (MRC) technique at the receiver. In Section III we review some STBC examples from [7], [9]-[11], emphasizing the code transmission matrix and the code parameters. Section IV shows simulations results and the performance comparison in terms of BER and diversity gain for different STBC codes chosen for a fixed number of transmit antennas. Finally, Section V draws some conclusions of this paper.

## II. A SPACE-TIME BLOCK CODED MIMO SYSTEM

We consider a general MIMO system with  $N_T$  transmit antennas and  $N_R$  receive antennas, employing a space-time encoder, a MIMO channel with  $N_T$  inputs and  $N_R$  outputs and a space-time decoder with MRC technique and Maximum Likelihood (ML) decoding.

Fig. 1 illustrates a simple example for two transmit and two receive antennas.

## A. The Space-Time Encoder

The first and the simplest transmit diversity scheme for two transmit antennas is the Alamouti code [6], described by the transmission matrix:

$$\mathbf{G} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}. \tag{1}$$

To transmit *m* bits/channel use we use a modulation that maps every *m* bits to one symbol from a real or complex constellation with  $M = 2^m$  symbols, for example PSK or QAM. The transmitter picks two symbols from the constellation, for example,  $x_1$  and  $x_2$ . In the first time slot  $t_1$ , the first antenna transmits the symbol  $x_1$  and the second antenna the symbol  $x_2$ . Then, in the second time slot  $t_2$ , the symbols  $-x_2^*$  and  $x_1^*$  are transmitted simultaneously from the two antennas. Both symbols  $x_1$  and  $x_2$  are spread over two transmit antennas and over two time slots (see Fig. 1).

At the transmitter, we do not know the channel (without CSIT), so we suppose an equal transmit power for each antenna and a unitary total transmit power.

## B. The Space-Time Coded MIMO Channel

The transmitted symbols over the MIMO channel are affected by severe magnitude fluctuations and phase rotations.



Fig. 1. A particular MIMO 2x2 system with Alamouti coding and MRC&ML decoding.

For two receive antennas, the received symbols are [6]:

$$y_{11} = h_{11}x_1 + h_{12}x_2 + n_{11}, \quad y_{12} = -h_{11}x_2^* + h_{12}x_1^* + n_{12}, y_{21} = h_{21}x_1 + h_{22}x_2 + n_{21}, \quad y_{22} = -h_{21}x_2^* + h_{22}x_1^* + n_{22},$$
(2)

where  $h_{ij}$  is the path gain between the  $j^{th}$  transmit antenna and the  $i^{th}$  receive antenna. The term  $n_{ij}$  is the additive noise for the  $i^{th}$  receive antenna at the  $j^{th}$  time slot, modeled as independent complex Gaussian random variables with zero-mean and variance 1/(2SNR) per complex dimension, where *SNR* is the signal to noise ratio of the channel.

We assume a quasi-static flat fading Rayleigh channel, with coherence time  $T_c$ . For a flat fading channel, the fading coefficients  $h_{ij}$  remain constant within a frame of length  $T_c$  time slots and change into new ones from frame to frame. Also, we assume uncorrelated path gains (the distance between two antennas is more that half of the wavelength) which vary independently from one frame to another. For a quasi-static channel, the path gains are constant over a frame of length multiple of  $T_c$ . For a Rayleigh channel, the path gains are independent complex Gaussian random variables, with zero mean and variance 0.5 per real dimension.

If the symbols  $y_{12}$  and  $y_{22}$  from equations (2) are complex conjugated, then we have:

$$y_{11} = h_{11}x_1 + h_{12}x_2 + n_{11}, \quad y_{12}^* = h_{12}^*x_1 - h_{11}^*x_2 + n_{12}^*, y_{21} = h_{21}x_1 + h_{22}x_2 + n_{21}, \quad y_{22}^* = h_{22}^*x_1 - h_{21}^*x_2 + n_{22}^*,$$
(3)

or in matrix form:

$$\begin{bmatrix} y_{11} \\ y_{21} \\ y_{12}^{*} \\ y_{22}^{*} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{12}^{*} & -h_{11}^{*} \\ h_{22}^{*} & -h_{21}^{*} \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \end{bmatrix} + \begin{bmatrix} n_{11} \\ n_{21} \\ n_{12}^{*} \\ n_{22}^{*} \end{bmatrix},$$
(4)

which can be rewritten:

$$\mathbf{y} = \mathbf{H}_{ef} \mathbf{x} + \mathbf{n} \ . \tag{5}$$

This relation represents the transfer function between the input **x** of the STBC encoder and the output **y** of the MIMO channel, where  $\mathbf{H}_{ef}$  is the matrix of the equivalent channel formed by the ST encoder and the MIMO channel. Moreover,  $\mathbf{H}_{ef}$  is an orthogonal matrix over all channel realizations because  $\mathbf{H}_{ef}^{H}\mathbf{H}_{ef} = \|\mathbf{H}\|_{F}^{2}\mathbf{I}_{2}$ , where  $\mathbf{H} = [h_{ij}]$  is the channel matrix and  $\|\cdot\|_{F}$  is the Frobenius norm.

# C. The Space-Time Decoder

At the receiver, we suppose a perfect CSIR, so we use the Maximal Ratio Combining (MRC) technique, combining coefficients being optimally chosen equal with the complex conjugated equivalent channel matrix [9]-[11]:

$$\tilde{\mathbf{x}} = \mathbf{H}_{ef}^{H} \mathbf{y} \,, \tag{6}$$

which can be detailed as:

$$\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} = \begin{bmatrix} h_{11}^* & h_{21}^* & h_{12} & h_{22} \\ h_{12}^* & h_{22}^* & -h_{11} & -h_{21} \end{bmatrix} \begin{bmatrix} y_{11} \\ y_{21} \\ y_{12}^* \\ y_{22}^* \end{bmatrix},$$
(7)

or:

$$\tilde{x}_{1} = h_{11}^{*} y_{11} + h_{12} y_{12}^{*} + h_{21}^{*} y_{21} + h_{22} y_{22}^{*},$$

$$\tilde{x}_{2} = h_{12}^{*} y_{11} - h_{11} y_{12}^{*} + h_{22}^{*} y_{21} - h_{21} y_{22}^{*}.$$
(8)

Finally, the combined symbols  $\tilde{x}_1$  and  $\tilde{x}_2$  are applied to a classical Maximum-Likelihood (ML) decoder to obtain reliable estimates of the transmitted symbols. Even if a path is severely faded, we may still be able to recover the transmitted symbols through other propagation paths.

Using (5), equation (6) can be written as:

$$\tilde{\mathbf{x}} = \mathbf{H}_{ef}^{H} \mathbf{H}_{ef} \mathbf{x} + \mathbf{H}_{ef}^{H} \mathbf{n} = \left\| \mathbf{H} \right\|_{F}^{2} \mathbf{I}_{2} \mathbf{x} + \mathbf{n}', \qquad (9)$$

where **n**' has zero-mean and autocorrelation function  $E[\mathbf{n'n'}^H] = \sigma_n^2 \|\mathbf{H}\|_F^2 \mathbf{I}_2$ . In this manner, the combined symbols become decoupled:

$$\tilde{x}_1 = \left\|\mathbf{H}\right\|_F^2 x_1 + \tilde{n}_1, \qquad \tilde{x}_2 = \left\|\mathbf{H}\right\|_F^2 x_2 + \tilde{n}_2,$$
 (10)

the symbol  $\tilde{x}_1$  being a function only of  $x_1$  and  $\tilde{x}_2$  only of  $x_2$ , this being the reason for which we can use ML decoding.

The MRC technique can be generalized easily for an arbitrary number  $N_R$  of receive antennas, including also one antenna case. The combined symbols are:

$$\tilde{x}_{1} = \sum_{i=1}^{N_{R}} \left( h_{i1}^{*} y_{i1} + h_{i2} y_{i2}^{*} \right), \qquad \tilde{x}_{2} = \sum_{i=1}^{N_{R}} \left( h_{i2}^{*} y_{i1} - h_{i1} y_{i2}^{*} \right).$$
(11)

### III. EXAMPLES OF SPACE-TIME BLOCK CODES

To generalize the Alamouti code for more than two transmit antennas, firstly, we have to make some remarks and give some definitions.

At the input of the ST encoder we consider a kdimensional vector  $\mathbf{x} = [x_1 \ x_2 \ \dots \ x_k]^T$  with elements from a certain complex signal constellation. At the output, we obtain the matrix codeword  $\mathbf{G} = [g_{ij}]$  of size  $T \times N_T$ , whose elements are combinations of the input complex symbols and their conjugates. During each i<sup>th</sup> time slot, the symbols  $g_{ij}$  are sent simultaneously from the transmit antennas j =1, 2, ...  $N_T$ . For each  $j^{\text{th}}$  transmit antenna, symbols  $g_{ij}$  are sent successively at i = 1, 2, ..., T time slots. Because we transmit k symbols from  $N_T$  antennas during T time slots, the code rate is R = k/T, the code being noted as  $C(N_T,k,T)$ . With this notation, the Alamouti code can be noted as C(2,2,2).

In practice, we are interested in particular codes, called Orthogonal Space Time Block Code (OSTBC), for which the transmission matrix G satisfies the conditions [6]:

- *linearity*: all elements  $g_{ii}$  are linear combinations of the input symbols and their conjugates;
- orthogonality:

$$\mathbf{G}^{H}\mathbf{G} = \|\mathbf{x}\|^{2} \mathbf{I}_{N_{T}} = \left(|x_{1}|^{2} + |x_{2}|^{2} + \dots + |x_{k}|^{2}\right) \mathbf{I}_{N_{T}}, \quad (12)$$

where  $\|\cdot\|$  is the Euclidian norm and  $\mathbf{I}_{N_T}$  is the  $N_T \times N_T$ identity matrix. These codes provide full diversity equal with  $N_T N_R$ , but have very little or no coding gain.

The algebraic theory that underlies the design of the orthogonal matrices according to (12) is the Hurwitz-Radon (HR) theory [8] that shows that a HR family with *N*-1 matrices of size NxN exists only if N = 2, 4 or 8.

Based on HR family of matrices the STBC transmission matrix can be constructed. For real constellations, square STBC matrices of full rate (R = 1) exist only for  $N_T = 2, 4$ or 8. However, non-square full rate code matrices can be constructed for other values of  $N_T = 3$ , 5, 6 and 7 by removing certain columns from square full rate matrices. For complex constellations, it is observed that the only value of  $N_T$  for which a full rate orthogonal STBC exists is  $N_T = 2$ . So, the construction of the Alamouti code cannot be extended to more than two transmit antennas. But, for  $N_T > 2$ , rate 1/2codes for complex constellations can be constructed from full rate codes for real constellations, so we must sacrifice the code rate to obtain an orthogonal design [7].

For  $N_T = 4$  transmit antennas, Tarokh et al. proposed in [7] the 1/2 rate code C(4,4,8):

$$\mathbf{G}_{2} = \begin{bmatrix} x_{1} & -x_{2} & -x_{3} & -x_{4} & x_{1}^{*} & -x_{2}^{*} & -x_{3}^{*} & -x_{4}^{*} \\ x_{2} & x_{1} & x_{4} & -x_{3} & x_{2}^{*} & x_{1}^{*} & x_{4}^{*} & -x_{3}^{*} \\ x_{3} & -x_{4} & x_{1} & x_{2} & x_{3}^{*} & -x_{4}^{*} & x_{1}^{*} & x_{2}^{*} \\ x_{4} & x_{3} & -x_{2} & x_{1} & x_{4}^{*} & x_{3}^{*} & -x_{2}^{*} & x_{1}^{*} \end{bmatrix}^{\mathsf{I}}, (13)$$

where the transpose operator is used to save space and is not related with the definition of transmission matrix G.

To increase the bandwidth efficiency, compared with 1/2 rate codes, in [7] are also defined codes of rate 3/4. An example is the code C(4,3,4):

$$\mathbf{G}_{3} = \begin{bmatrix} x_{1} & x_{2} & x_{3} & 0 \\ -x_{2}^{*} & x_{1}^{*} & 0 & x_{3} \\ x_{3}^{*} & 0 & -x_{1}^{*} & x_{2} \\ 0 & x_{3}^{*} & -x_{2}^{*} & -x_{1} \end{bmatrix}.$$
 (14)

If we remove the symbol  $x_3$ , we maintain the orthogonal structure of the matrix, therefore, a new code of rate 1/2can be obtained, the C(4,2,4):

$$\mathbf{G}_{4} = \begin{bmatrix} x_{1} & x_{2} & 0 & 0 \\ -x_{2}^{*} & x_{1}^{*} & 0 & 0 \\ 0 & 0 & -x_{1}^{*} & x_{2} \\ 0 & 0 & -x_{2}^{*} & -x_{1} \end{bmatrix}.$$
 (15)

Another code of interest is C(4,4,4), presented in [10]:

$$\mathbf{G}_{5} = \begin{bmatrix} x_{1} & x_{2} & x_{3} & x_{4} \\ -x_{2}^{*} & x_{1}^{*} & -x_{4}^{*} & x_{3}^{*} \\ x_{3} & x_{4} & x_{1} & x_{2} \\ -x_{4}^{*} & x_{3}^{*} & -x_{2}^{*} & x_{1}^{*} \end{bmatrix},$$
(16)

which is a Quasi-Orthogonal Space-Time Block Code (OOSTBC) of rate 1 and with a second-order diversity.

For  $N_T = 8$  transmit antennas, the code C(8,8,16) of rate 1/2 given in [7] is:

$$\mathbf{G}_{6} = \begin{bmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} & x_{8} \\ -x_{2} & x_{1} & x_{4} & -x_{3} & x_{6} & -x_{5} & -x_{8} & x_{7} \\ -x_{3} & -x_{4} & x_{1} & x_{2} & x_{7} & x_{8} & -x_{5} & -x_{6} \\ -x_{4} & x_{3} & -x_{2} & x_{1} & x_{8} & -x_{7} & x_{6} & -x_{5} \\ -x_{5} & -x_{6} & -x_{7} & -x_{8} & x_{1} & x_{2} & x_{3} & x_{4} \\ -x_{6} & x_{5} & -x_{8} & x_{7} & -x_{2} & x_{1} & -x_{4} & x_{3} \\ -x_{7} & x_{8} & x_{5} & -x_{6} & -x_{3} & x_{4} & x_{1} & -x_{2} \\ -x_{8} & -x_{7} & x_{6} & x_{5} & -x_{4} & -x_{3} & x_{2} & x_{1} \\ x_{1}^{*} & x_{2}^{*} & x_{3}^{*} & x_{4}^{*} & x_{5}^{*} & x_{6}^{*} & x_{7}^{*} & x_{8}^{*} \\ -x_{2}^{*} & x_{1}^{*} & x_{4}^{*} & -x_{3}^{*} & x_{6}^{*} & -x_{5}^{*} & -x_{6}^{*} \\ -x_{4}^{*} & x_{3}^{*} & -x_{2}^{*} & x_{1}^{*} & x_{8}^{*} & -x_{5}^{*} & -x_{6}^{*} \\ -x_{6}^{*} & x_{5}^{*} & -x_{8}^{*} & x_{7}^{*} & -x_{8}^{*} & x_{1}^{*} & -x_{2}^{*} & x_{3}^{*} & x_{4}^{*} \\ -x_{6}^{*} & x_{5}^{*} & -x_{8}^{*} & x_{7}^{*} & -x_{2}^{*} & x_{1}^{*} & -x_{4}^{*} & x_{3}^{*} \\ -x_{6}^{*} & x_{5}^{*} & -x_{6}^{*} & -x_{3}^{*} & x_{4}^{*} & x_{1}^{*} & -x_{2}^{*} \\ -x_{8}^{*} & -x_{7}^{*} & x_{6}^{*} & x_{5}^{*} & -x_{4}^{*} & -x_{3}^{*} & x_{2}^{*} & x_{1}^{*} \end{bmatrix} \right]$$
Another 1/2 rate code defined in [9] is C(8,4,8):
$$\begin{bmatrix} x_{1} & 0 & 0 & 0 & -x_{4}^{*} & 0 & -x_{2}^{*} & x_{3}^{*} \\ 0 & x & 0 & 0 & 0 & -x_{4}^{*} & 0 & -x_{2}^{*} & x_{3}^{*} \end{bmatrix}$$

$$\mathbf{G}_{7} = \begin{bmatrix} x_{1} & 0 & 0 & 0 & -x_{4}^{*} & 0 & -x_{2}^{*} & x_{3}^{*} \\ 0 & x_{1} & 0 & 0 & 0 & -x_{4}^{*} & -x_{3} & -x_{2} \\ 0 & 0 & x_{1} & 0 & x_{2} & x_{3}^{*} & -x_{4} & 0 \\ 0 & 0 & 0 & x_{1} & -x_{3} & x_{2}^{*} & 0 & -x_{4} \\ x_{4} & 0 & -x_{2}^{*} & x_{3}^{*} & x_{1}^{*} & 0 & 0 & 0 \\ 0 & x_{4} & -x_{3} & -x_{2} & 0 & x_{1}^{*} & 0 & 0 \\ x_{2} & x_{3}^{*} & x_{4}^{*} & 0 & 0 & 0 & x_{1}^{*} & 0 \\ -x_{3} & x_{2}^{*} & 0 & x_{4}^{*} & 0 & 0 & 0 & x_{1}^{*} \end{bmatrix}.$$
 (18)

To obtain codes for other numbers of transmit antennas, for example  $N_T = 3, 5, 6$  or 7, we can eliminate one or more columns from the transmission matrix of the previous codes, the columns orthogonality remaining the same.

0

For  $N_T = 3$  transmit antennas, we can remove the fourth column from  $G_2$  and we obtain the code C(3,4,8) [7]:

$$\mathbf{G}_{8} = \begin{bmatrix} x_{1} & -x_{2} & -x_{3} & -x_{4} & x_{1}^{*} & -x_{2}^{*} & -x_{3}^{*} & -x_{4}^{*} \\ x_{2} & x_{1} & x_{4} & -x_{3} & x_{2}^{*} & x_{1}^{*} & x_{4}^{*} & -x_{3}^{*} \\ x_{3} & -x_{4} & x_{1} & x_{2} & x_{3}^{*} & -x_{4}^{*} & x_{1}^{*} & x_{2}^{*} \end{bmatrix}^{\mathrm{I}} . (19)$$

In the same manner, removing the fourth column from  $G_3$  we can obtain the code C(3,3,4) of rate 3/4:

$$\mathbf{G}_{9} = \begin{bmatrix} x_{1} & x_{2} & x_{3} \\ -x_{2}^{*} & x_{1}^{*} & 0 \\ x_{3}^{*} & 0 & -x_{1}^{*} \\ 0 & x_{3}^{*} & -x_{2}^{*} \end{bmatrix},$$
(20)

and from  $G_4$  the code C(3,2,4) of rate 1/2:

$$\mathbf{G}_{10} = \begin{bmatrix} x_1 & x_2 & 0 \\ -x_2^* & x_1^* & 0 \\ 0 & 0 & -x_1^* \\ 0 & 0 & -x_2^* \end{bmatrix}.$$
 (21)

## IV. SIMULATION RESULTS

In this section, we provide and compare the simulation results for the implemented STBC-MIMO system using ten STBC codes over a quasi-static flat fading Rayleigh channel [12], [13]. For a bit rate of 1 bps/Hz we use BPSK and QPSK modulations for codes of rate 1 and 1/2. The channel coherence time is equal with the number of time slots,  $T_c=T$ , and the channel is quasi-static over 10*T* time slots. At the receiver, for each STBC code the equivalent channel matrix  $\mathbf{H}_{ef}$  defined in (5) is computed in order to determine the combined symbols  $\tilde{x}_1$  and  $\tilde{x}_2$  from (8). To evaluate the system performances the Bit Error Rate (BER) is measured and plotted versus the channel Signal to Noise Ratio (SNR). The simulation ends after the transmission of  $2x10^6$  symbols or after  $10^3$  errors were found.

First, we study the diversity gain for a variable number of transmit and/or receive antennas,  $N_T x N_R = 1x1$ , 2x1, 1x2 and 2x2, see Fig. 2. For two transmit antennas we use the Alamouti C(2,2,2) code and for one or two receive antennas the MRC technique. It can be seen that for 2x1, 1x2 and 2x2 systems, the diversity gains in SNR over 1x1system are 10 dB, 13 dB and 17 dB for BER= $10^{-3}$ . The 2x1 system performance is 3 dB worse than that of the 1x2system, even if both systems have the same diversity order  $N_T N_R$ =2. Because of this, we choose the 1x2 system with  $N_R$ =2 receive antennas and MRC technique as a "reference uncoded system" for the following simulations. By considering this scenario, we can analyze how the performance of the MIMO system can be improved by changing only the STBC code at the transmitter side.

Second, to study the performances of different STBCs, we keep fixed the number of transmit antennas and we vary the STBC choosing a certain modulation to assure 1 bps/Hz.

For  $N_T = 3$  transmit antennas the performances of the C(3,4,8), C(3,3,4) and C(3,2,4) are shown in Fig. 3 for QPSK. It is seen that the first two codes have a diversity gain of 11.5 dB at BER=10<sup>-5</sup>, which is by 3 dB better than the diversity gain of the third code. But, the second code of rate 3/4 gives us a higher 1.5 bps/Hz bit rate.

For  $N_T = 4$  transmit antennas the performances of the C(4,4,8), C(4,2,4), C(4,4,4) and C(4,3,4) are shown in Fig. 4. For full and 1/2 rate codes we use BPSK and QPSK to assure 1 bps. For 3/4 rate code we use QPSK, resulting in a 1.5 bps/Hz bit rate. The best codes are the first and the



Fig. 2. BER (SNR) of Alamouti code and MRC for a variable number of transmit/receive antennas.



Fig. 3. BER (SNR) of different STBC for  $N_T = 3$  antennas.



Fig. 4. BER (SNR) of different STBC for  $N_T = 4$  antennas.

fourth, which assures a diversity gain of 12.5 dB at BER =  $10^{-5}$ , the intermediate code is C(4,2,4) with 6.5 dB and the worst code is C(4,4,4) with no diversity gain.

For  $N_T = 8$  transmit antennas the performances of the C(8,8,16) and C(8,4,8) with QPSK are shown in Fig. 5. As we can see, the first code is better, with a diversity gain of 14 dB at BER =  $10^{-5}$  and the second code has only 11 dB gain assured by the use of multiple transmit antennas.



Fig. 5. BER (SNR) of different STBC for  $N_T = 8$  antennas.



Fig. 6. BER (SNR) of the best STBC for  $N_T = 1, 2, 3, 4$  or 8 transmit antennas.

Finally, Fig. 6 provides a performance comparison for 1 bps/Hz STBC-MIMO system using  $N_T = 1$  (uncoded), 2, 3, 4 and 8 transmit antennas for which we choose the best codes from previous simulations. The best performance is obtained for the maximum number of transmit antennas. For a BER = 10<sup>-5</sup>, if the number  $N_T$  of transmit antennas is increased from 1 to 2, from 2 to 3, from 3 to 4 and from 4 to 8, the additional diversity gains are 9 dB, 2.5 dB, 1 dB and 1.5 dB. This is so because much of the diversity gain is already achieved using two transmit and two receive antennas.

These simulations show that significant gains can be achieved by increasing the number of transmit antennas. If the 1 bps/Hz bit rate is increased by using a higher order modulation scheme, the STBC-MIMO system will suffer some performance degradation, because a high order constellation is more densely and it is more sensitive to errors than a low order constellation. Moreover, the analyzed STBC codes have no error correction capabilities.

# V. CONCLUSION

In this paper, we have reviewed the encoding and decoding process for the Alamouti code. For other nine different STBC codes from the literature, we have specified the transmission matrix and the code parameters. Then, we have provided simulation results to compare the performances of different STBC codes chosen for a fixed number of transmit antennas, specifying the best code that assures a maximum diversity gain and a minimum BER. Also, significant gains can be achieved by increasing the number of transmit antennas. However, the concatenation of STBC codes with classical channel codes like convolutional or turbo codes, can offer optimal diversity and coding gain, with the expense of a decreased bit rate and an increased complexity.

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