A Comparison of Different Engineering Models for Computation of Lightning Magnetic Field of Negative First Strokes

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***Abstract* — A comparison of different engineering models results for a lightning magnetic field of negative first strokes is presented in this paper. A new function for representing double-peaked channel-base current is used for lightning stroke modeling. This function includes the initial and subsidiary peak in a current waveform. For experimentally measured currents, a magnetic field is calculated for the three engineering models: transmission line (TL) model, TL model with linear decay (MTLL), and TL model with exponential decay (MTLE).**

***Keywords* — Channel-base current, electromagnetic field, lightning discharge, lightning stroke model, return stroke.**

I. Introduction

LIGHTNING stroke modeling is an important issue to be solved in order to estimate vulnerability of electric systems, electronic devices and equipment in a lightning electromagnetic field (LEMF). In the case of some existing protective or shielding structure its lightning protection level (LPL) should be estimated and defined [1].

A comparison of measured LEMF components and results obtained using different engineering models points to their shortages and advantages. Simulations using computer programs are preferable in comparison to cumbersome measurements and purchase of expensive laboratory equipment. Thus, any improvement in electromagnetic modeling of lightning strokes is useful, especially if suitable to be included in already designed computer programs. This paper presents an improvement in modeling of negative first strokes currents.

A channel-base current with the initial and subsidiary peak is approximated with the two-rise front function (TRF) proposed already in [2]-[4]. Calculated LEMF waveforms differ from results obtained with one-peaked current which is usually used in literature. Results obtained with TRF are in better agreement with experimental results [5]-[7]. Different waveshapes of channel-base currents implied in electromagnetic, engineering or transmission line models make great influence [8] on LEMF waveforms.

There is an overview and classification of usually used lightning stroke models presented in [9]. LEMF results for different models of lightning strokes are compared in many papers, but the influence of different channel-base currents has not been investigated. Some of the models do and others don't provide main LEMF features as listed in [9]-[11]. Based on experimental results [5]-[7], these are: 1) a sharp initial peak in both electric and magnetic fields measured beyond a km or so; 2) a slow ramp following the initial peak for electric fields measured within a few tens of km; 3) a hump following the initial peak in magnetic fields measured within several tens of km; 4) zero crossing within tens of μs from the initial peak in both electric and magnetic fields beyond about 50km; and 5) a characteristic flattening of vertical electric field in about 15μs at tens to hundreds of meters.

It can be noticed from Table 1, adapted from [10] and [11], that transmission line (TL) model doesn't provide feature 2); modified transmission line model with linear decay (MTLL), modified TL model with exponential decay (MTLE), and modified TL model with current distortion (MTLD) don't provide feature 3); TL, traveling-current-source model (TCS), and Diendorfer-Uman model (DU) don't perform zero crossing 4); TL and MTLE don't provide feature 5). However, all these remarks are given for one-peaked channel-base currents results. If a double-peaked current is used instead as for experimentally measured negative first strokes currents, some of the models' shortages are overcome.

TABLE 1: ENGINEERING MODELS EVALUATED BY BENCHMARK.

Models	***1) Initial peak in close E- and H-field***	***2) E-field ramp at a few km***	***3) H-field hump at a few km***	***4) E- and H-field zero crossing at 100 km***	***5) E-field flattening at tens of meters***
TL	Yes	No	Yes	No	No
MTLL	Yes	Yes	No	Yes	Yes
MTLE	Yes	Yes	No	Yes	No
MTLD	Yes	Yes	No	Yes	Yes
TCS	Yes	Yes	Yes	No	Yes
DU	Yes	Yes	Yes	No	Yes

Adapted from Baba et al. [10], 2004, and Nucci et al. [11], 1990.

Besides the one-peaked Heidler's function [12], which is also used in IEC 62305 Std. [1] for typical lightning strokes, there were attempts to obtain double-peaked functions as sums of a few Heidler's functions. In [13], as an example, two Heidler's functions were summarized, but seven were needed in [14] for representing measured lightning currents [5], [7], and such an obtained function

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was used for calculations of lightning overvoltages in [15]. TRF function given here has fewer parameters and is simpler to be used for approximation of the same lightning currents ([5] and [7]).

II. Transmission Line Models with or without Decay as Engineering Models of Lightning Strokes

An engineering model of a lightning stroke assumes that the current along the channel depends on the height z' above the ground and decays from the channel-base current $i(0,t)$ according to the equation:

$$i(z',t) = u(t - z'/v_f)P(z',t)i(0, t - z'/v), \qquad (1)$$

where $u(t)$ is the Heaviside's function, v_f is the return-stroke speed, v is the current-wave propagation speed, and $P(z',t)$ is the height- and time-dependent current attenuation factor. In TL model this factor is $P(z',t)=1$, in MTLE it is $P(z',t)=\exp(-z'/\lambda)$, for the decaying constant λ, and in MTLL it is $P(z',t)=1-z'/H$, for the channel height H. Other engineering models such as MTLD, TCS, DU, Bruce-Golde (BG) model etc., are not discussed here. Lightning magnetic field is calculated for the three models: TL, MTLE and MTLL.

In the case of a vertical lightning current channel electric field has only a vertical component at the perfectly conducting ground surface, and magnetic field only an azimuthal component. It can be calculated based on antenna theory relations, using expression from [16]:

$$H_\psi(\vec{R},t) = \frac{1}{4\pi}\int_{-H}^{H}\left[\frac{r}{R_k^{\,3}}\, i\left(z', t-\frac{R_k}{\mathrm{c}}\right) + \frac{r}{\mathrm{c}\,R_k^{\,2}}\,\frac{\partial i\left(z', t-\frac{R_k}{\mathrm{c}}\right)}{\partial t}\right]\mathrm{d}z', \qquad (2)$$

or ε_0 - the permittivity of the air, μ_0 - the permeability of the air, $\mathrm{c}=(\varepsilon_0\mu_0)^{-1/2}$ - the speed of light, R_k - the distance from the current element (k=1), or its image in plane mirror (k=2) to the point P(r, ψ, z), as presented in Fig.1.

III. Double-Peaked Function for Representing Channel Base Current

TRF function with a two-rise front is given with the following expression [2], [3]:

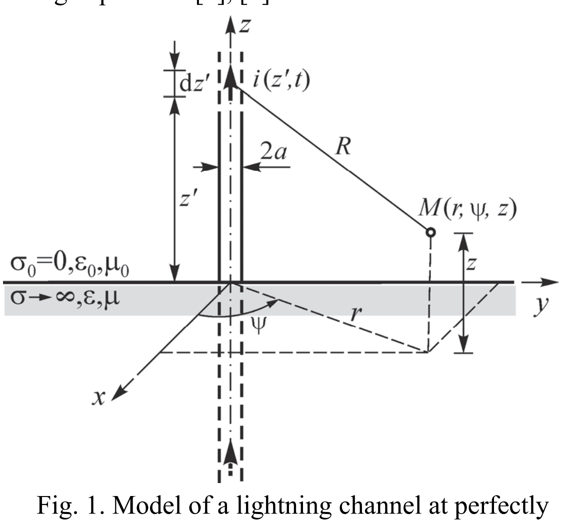

Fig. 1. Model of a lightning channel at perfectly conducting ground.

$$i(0,t) = \begin{cases} I_{m1}\sum_{i=1}^{k} d_i\left[\frac{t}{t_{m1}}\exp\left(1-\frac{t}{t_{m1}}\right)\right]^{a_i}, & 0 \le t \le t_{m1}, \\ I_{m1} + I_{m2}\sum_{i=1}^{l} f_i\left[\frac{t-t_{m1}}{t_{m2}-t_{m1}}\exp\left(1-\frac{t-t_{m1}}{t_{m2}-t_{m1}}\right)\right]^{b_i}, & t_{m1} \le t \le t_{m2}, \\ (I_{m1}+I_{m2})\sum_{i=1}^{n} g_i\left[\frac{t}{t_{m2}}\exp\left(1-\frac{t}{t_{m2}}\right)\right]^{c_i}, & t_{m2} \le t < \infty, \end{cases} \qquad (3)$$

for parameters a_i, b_i and c_i. The weighting coefficients d_i, f_i, and g_i are satisfying the condition:

$$\sum_{1}^{k} d_i = \sum_{1}^{l} f_i = \sum_{1}^{n} g_i = 1, \qquad (4)$$

k is the number of terms in the first rising part, l in the second rising part, and n in the decaying part (Fig. 2). I_{m1} and I_{m2} are two maximum current values of the function, in time moments t_{m1} and t_{m2}, respectively. The first derivative of the function is determined analytically, as the following:

$$\frac{\mathrm{d}i(t)}{\mathrm{d}t} = \begin{cases} I_{m1}t_{m1}^{-1}\sum_{i=1}^{k} d_i a_i\left[\left(\frac{t}{t_{m1}}\right)^{a_i-1} - \left(\frac{t}{t_{m1}}\right)^{a_i}\right]\exp\left[a_i\left(1-\frac{t}{t_{m1}}\right)\right], & 0 \le t \le t_{m1}, \\ I_{m2}(t_{m2}-t_{m1})^{-1}\sum_{i=1}^{l} f_i b_i\left[\left(\frac{t-t_{m1}}{t_{m2}-t_{m1}}\right)^{b_i-1} - \left(\frac{t-t_{m1}}{t_{m2}-t_{m1}}\right)^{b_i}\right]\exp\left[b_i\left(1-\frac{t-t_{m1}}{t_{m2}-t_{m1}}\right)\right], & t_{m1} \le t \le t_{m2}, \\ (I_{m1}+I_{m2})t_{m2}^{-1}\sum_{i=1}^{n} g_i c_i\left[\left(\frac{t}{t_{m2}}\right)^{c_i-1} - \left(\frac{t}{t_{m2}}\right)^{c_i}\right]\exp\left[c_i\left(1-\frac{t}{t_{m2}}\right)\right], & t_{m2} \le t < \infty. \end{cases} \qquad (5)$$

TRF is used for approximation of channel-base currents in engineering models given with (1). It can represent currents of negative first stroke channel-base currents as experimentally measured at Monte San Salvatore in Switzerland [5], denoted with MSS_FST#2peaks in [14], and those measured at Morro do Cashimbo Station in Brazil [7], denoted with MCS_FST#2peaks in [14]. For approximating these two with TRF, its parameters are given in Table 2, and the corresponding functions, adequate to given in [14], are presented in Figs. 2 and 3.

Table 2: Parameters of TRF function representing MSS_FST#2peaks and MCS_FST#2peaks [14].

TRF parameters	**MSS_FST#2peaks**	**MCS_FST#2peaks**
I_{m1} (kA)	27.66	40.07
I_{m2} (kA)	3.34	5.215
t_{m1} (μs)	8.2	8.2
t_{m2} (μs)	13.6	13.8
d_1	0.37	0.37
d_2 =1- d_1	0.63	0.63
a_1	2.2	2.2
a_2	28	28
f_1	1	1
b_1	5.5	15
g_1	0.4	0.45
g_2=1- g_1	0.6	0.55
c_1	2	3.3
c_2	0.06	0.055

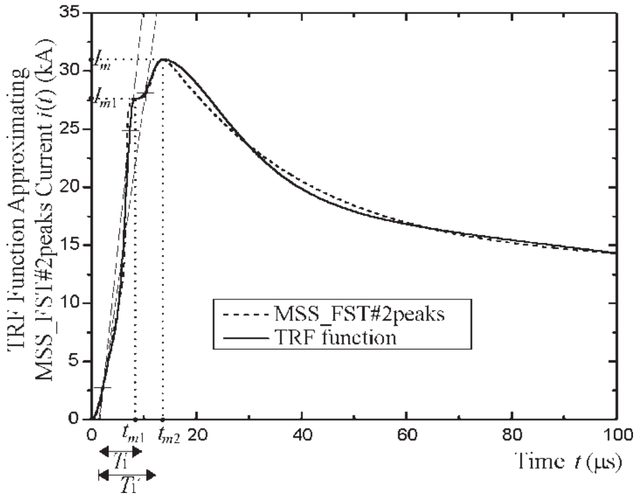

Fig. 2. TRF representing MSS_FST#2peaks current.

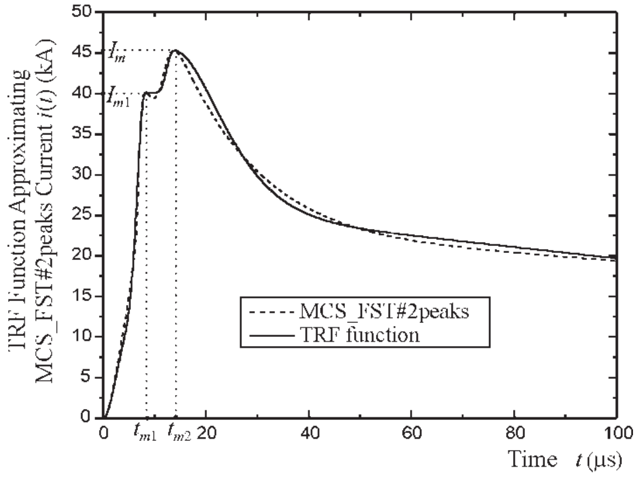

Fig. 3. TRF representing MCS_FST#2peaks current.

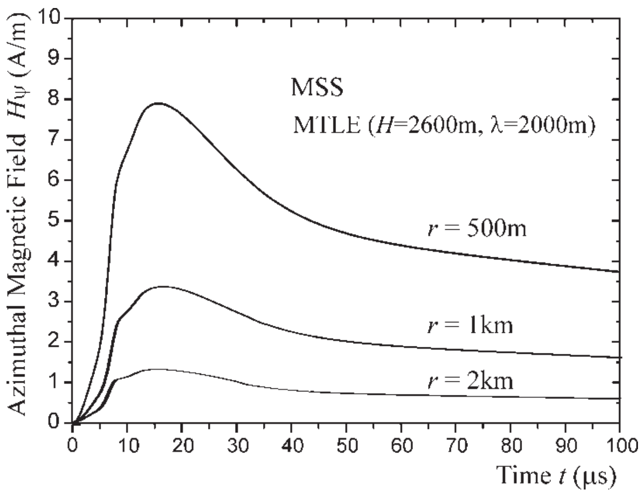

Fig. 4. Azimuthal magnetic field for MSS_FST#2peaks current calculated by MTLE for r=500m, 1km and 2km from the channel-base, at the ground surface z=0.

IV. Magnetic Field Results for Modified Transmission Line Model with Exponential Decay

Results for the azimuthal component of magnetic field at perfectly conducting ground are calculated for engineering model MTLE and presented in Figs. 4-6.

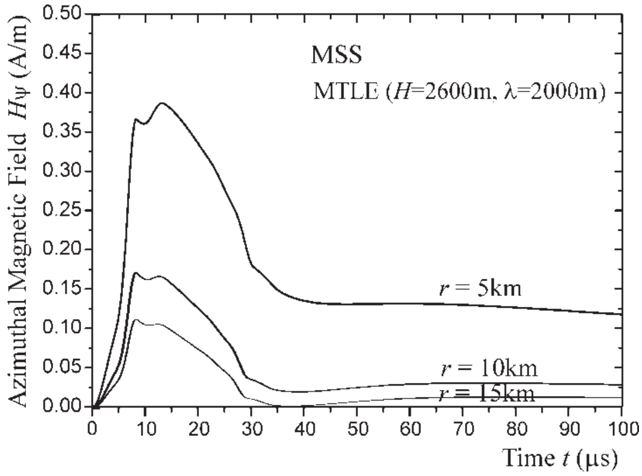

Fig. 5. Azimuthal magnetic field for MSS_FST#2peaks current calculated by MTLE for r=5km, 10km and 15km from the channel-base, at the ground surface z=0.

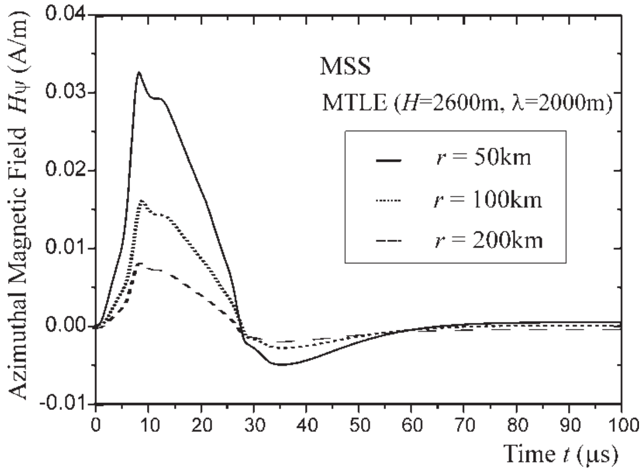

Fig. 6. Azimuthal magnetic field for MSS_FST#2peaks current calculated by MTLE for r=50km, 100km and 200km from the channel-base, at the ground surface z=0.

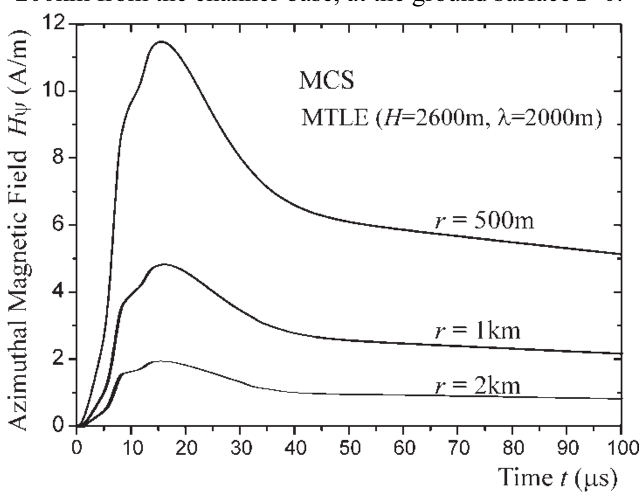

Fig. 7. Azimuthal magnetic field for MCS_FST#2peaks current calculated by MTLE for r=500m, 1km and 2km from the channel-base, at the ground surface z=0.

For MSS_FST#2peaks current as in Fig. 2, results are calculated for the lightning channel height H=2600m, v=v_f=1.3$\cdot$10^{8}m/s, the constant λ=2000m, and distances ranging from 500m to 200km.

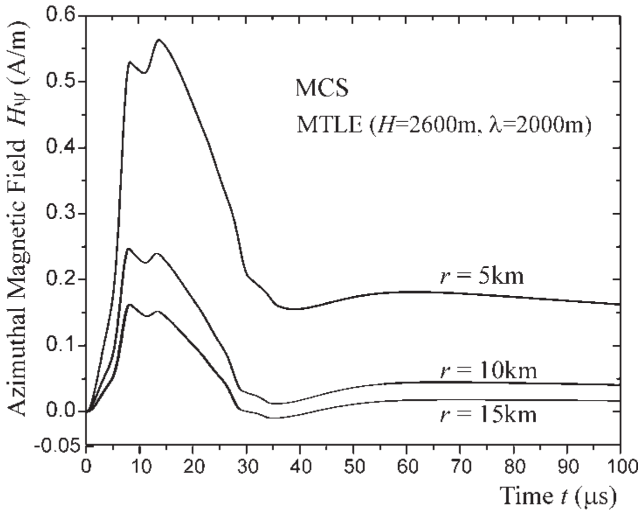

Fig. 8. Azimuthal magnetic field for MCS_FST#2peaks current calculated by MTLE for r=5km, 10km and 15km from the channel-base, at the ground surface z=0.

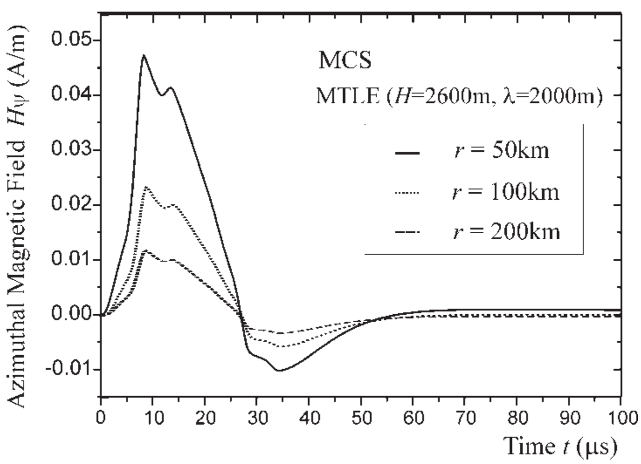

Fig. 9. Azimuthal magnetic field for MCS_FST#2peaks current calculated by MTLE for r=50km, 100km and 200km from the channel-base, at the ground surface z=0.

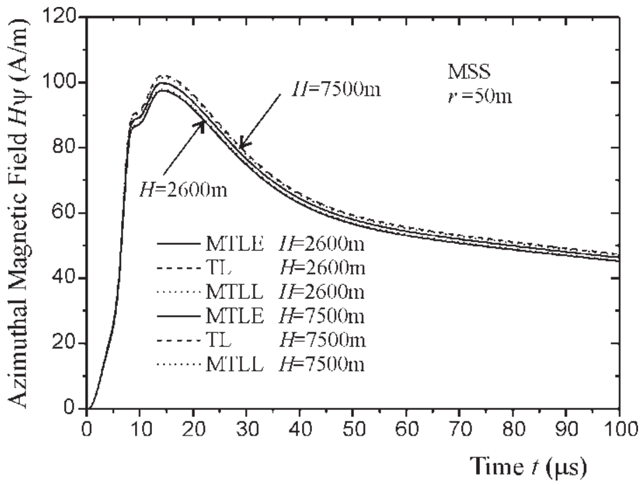

Fig. 10. Azimuthal magnetic field for the first stroke current MSS_FST#2peaks [5], for the three models, two channel heights H=2600m and 7500m, r=50m and z=0.

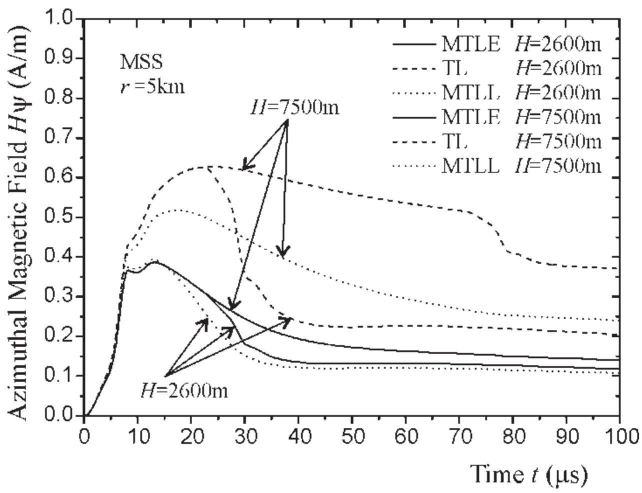

Fig. 11. Azimuthal magnetic field for the first stroke current MSS_FST#2peaks [5], for the three models, two channel heights H=2600m and 7500m, r=5km and z=0.

For MCS_FST#2peaks current, as given in Fig. 3, results for azimuthal magnetic field at perfectly conducting ground are calculated for the lightning channel height H=2600m, $v=v_f=1.3\cdot10^8$m/s, the constant λ=2000m, and presented in Fig. 7 for distances ranging from 500m to 2km, in Fig. 8 for 5-15km, and in Fig. 9 for 50-200km.

V. Magnetic Field Results for Different Engineering Models

Results for calculated azimuthal magnetic field at perfectly conducting ground are presented in Figs. 10-15 for two different lightning channel heights H=2600m and H=7500m, $v=v_f=1.3\cdot10^8$m/s in (1), for the three chosen engineering models: TL, MTLL and MTLE. The decaying constant is λ=2000m for MTLE, as in [11]. For the current MSS_FST#2peaks, as in Fig. 2, results are given in Figs. 10-12 for the distances r=50m, 5km and 100km from the channel-base, at the ground surface point for z=0. For the current MCS_FST#2peaks, as in Fig. 3, and the same distances, results are given in Figs. 13-15.

The following can be concluded from magnetic field results of these models at different distances from the channel-base:

a) Azimuthal magnetic field waveshape at the distance of 50m approximately follows the channel-base current waveshape for all the models and for different channel heights.
b) For all the models azimuthal magnetic field perform zero-crossing at 100km, including TL model. This was also noticed for vertical electric field results [4].
c) MTLE and MTLL have some sort of a hump at the distance of 5km which also depends on the channel-base current waveshape. However, for this distance, MTLL results show better agreement of the waveform with experimental results from [5].
d) The initial peak is not so emphasized due to the chosen waveshapes of currents with slower rising than in the case of usually used one-peaked current waveshape.

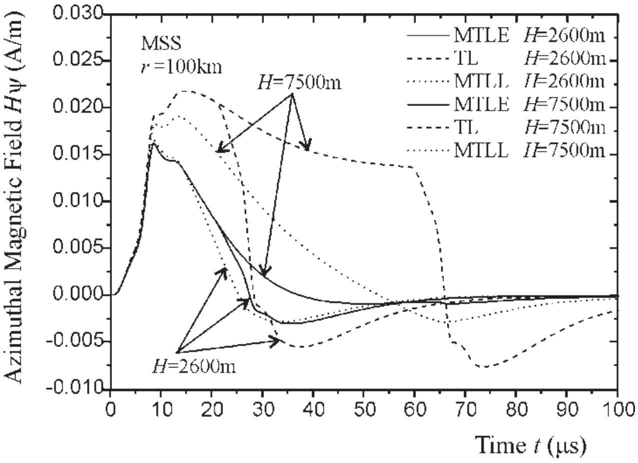

Fig. 12. Azimuthal magnetic field for the first stroke current MSS_FST#2peaks [5], for the three models, two channel heights H=2600m and 7500m, r=100km and z=0.

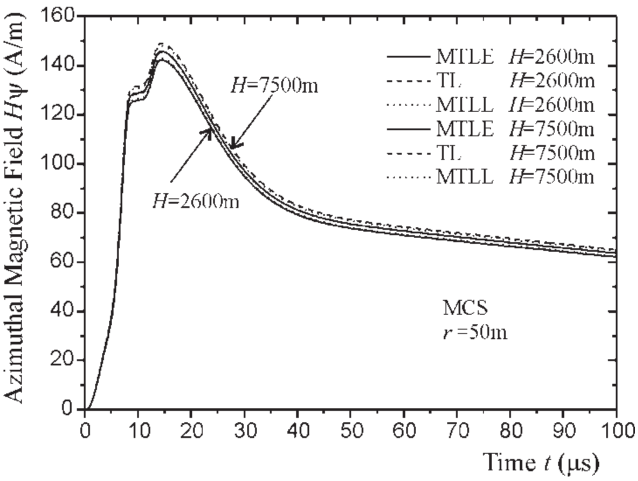

Fig. 13. Azimuthal magnetic field for the first stroke current MCS_FST#2peaks [7], for the three models, two channel heights H=2600m and 7500m, r=50m and z=0.

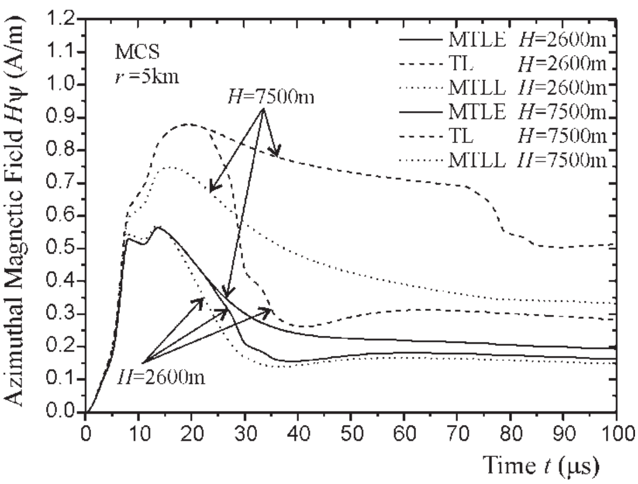

Fig. 14. Azimuthal magnetic field for the first stroke current MCS_FST#2peaks [7], for the three models, two channel heights H=2600m and 7500m, r=5km and z=0.

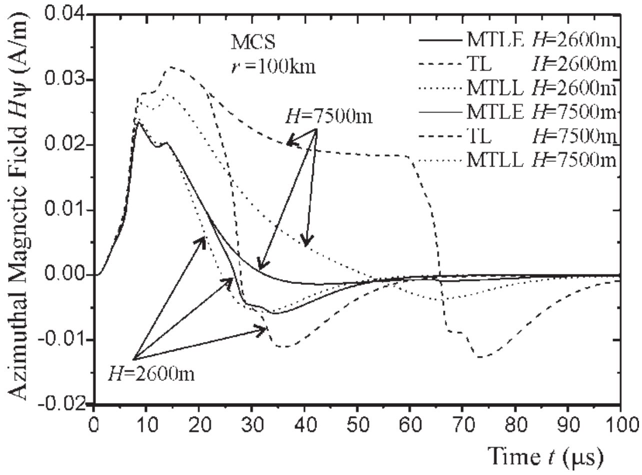

Fig. 15. Azimuthal magnetic field for the first stroke current MCS_FST#2peaks [7], for the three models, two channel heights H=2600m and 7500m, r=100km and z=0.

e) Lightning channel height has a greater influence on the waveshapes of azimuthal magnetic field obtained by using TL and MTLL than if MTLE is used.

f) MTLL gives better results than other two models if the double-peaked channel-base current is used.

VI. Conclusion

Although first negative return strokes have branching, these are not taken into account in presented models and it would be very complicated to do so for this natural phenomenon. Thus the analysis presented in this paper cannot give the answer about the overall validity of either model.

Double- and multi-peaked channel-base currents of negative first strokes were experimentally measured and investigated in literature, but the double-peaked function presented here used for current approximation and LEMP calculations could be more comfortable to use and easier for application and including in already designed computer programs than other functions from literature. Multi-peaked currents can be obtained with a similar function, also providing analytical derivative, integral, and integral transformations for calculations.

Results presented in this paper for an azimuthal magnetic field at different distances from the channel-base point to the great influence of channel-base current functions implied in models of lightning strokes.

Azimuthal magnetic field results obtained by using MTLE model are presented in a wide range of distances from the channel-base (from close to far field distances) using approximations of the measured negative first stroke currents. A comparison of lightning magnetic field results for three different engineering models is also presented. Lightning magnetic field results obtained by MTLL, for the channel-base current approximated with the double-peaked function, are in better agreement with experimental results [5] than if MTLE and TL models are used.

REFERENCES

[1] *IEC 62305 standard*, Protection against lightning – Part I: General principles, 2006.

[2] V. Javor, "Lightning electromagnetic field of a double-peaked channel-base current," in *CD Proceedings of papers, 19th Int. Conference on Software, Telecommunications & Computer Networks SoftCOM, Symp. on Environmental EMC*, Sep. 2011. http://ieeexplore.ieee.org/xpl/freeabs_all.jsp?arnumber=6064387

[3] V. Javor, "Approximation of a lightning channel-base current with a two-rise front," in *COMPEL, Emerald*, pp. 1007-1017, vol. 31, No. 3, 2012.

[4] V. Javor, "Modeling of lightning strokes using two-peaked channel-base currents," *International Journal of Antennas and Propagation*, Hindawi, doi: 10.1155/IJAP/318417, 2012. http://www.hindawi.com/journals/ijap/aip/318417

[5] Y. T. Lin, M. A. Uman, J. A. Tiller, R. D. Brantley, W. H. Beasley, E. P. Krider, and C. D. Weidman, "Characterization of lightning return stroke electric and magnetic fields from simultaneous two-station measurements," *In Journal of Geophys. Research*, vol. 84, pp. 6307-6314, 1979.

[6] K. Berger, R. B. Anderson, and H. Kroninger, "Parameters of lightning flashes," *Electra 80*, pp. 23-37, 1975.

[7] S. Visacro, A. Jr. Soares, M. A. O. Schroeder, L. C. L. Cherchiglia, and V. J. Souza, "Statistical analysis of lightning current parameters: Measurements at Morro do Cachimbo station," *Journal of Geophys. Research*, vol. 109, D01105, 2004.

[8] V. Javor and P. D. Rancic, "A channel-base current function for lightning return-stroke modelling," *IEEE Trans. Electromagn. Compat.*, vol. 53, no.1, pp. 245-249, 2011.

[9] V. A. Rakov and M. A. Uman, "Review and evaluation of lightning return stroke models including some aspects of their application," *IEEE Trans. Electromagn. Compat.*, vol. 40, no. 4, pp. 403-426, Nov. 1998.

[10] Y. Baba, S. Miyazaki, and M. Ishii, "Reproduction of lightning electromagnetic field waveforms by engineering model of return sroke," *IEEE Trans. Electromagn. Compat.*, vol. 46, no. 1, pp. 130-133, Feb. 2004.

[11] C. A. Nucci, G. Diendorfer, M. A. Uman, F. Rachidi, M. Ianoz, and C. Mazzetti, "Lightning return stroke current models with specified channel-base current: a review and comparison," *Journal of Geophys. Research*, vol. 95, no. D12, pp. 20395-408, 1990.

[12] F. Heidler, "Travelling current source model for LEMP calculation," *Proc. 6th Int. Zurich Symp. EMC*, Zurich, Switzerland, pp. 157-162, 1985.

[13] F. Heidler and J. Cvetic, "A class of analytical functions to study the lightning effects associated with the current front," *ETEP*, vol. 12, no. 2, pp. 141-150, 2002.

[14] A. De Conti and S. Visacro, "Analytical Representation of single- and double-peaked lightning current waveforms," *IEEE Trans. Electromagn. Compat.*, vol. 49, no. 2, pp. 448-451, 2007.

[15] F. H. Silveira, A. De Conti, and S. Visacro, "Lightning overvoltage due to first strokes considering a realistic current representation," *IEEE Trans. Electromagn. Compat.*, vol. 52, no. 4, pp. 929-935, 2010.

[16] M. J. Master and M. A. Uman, "Transient electric and magnetic field associated with establishing a finite electrostatic dipole," *American J. Phys.*, vol. 51, no. 2, pp. 118-126, Feb. 1983.