Design and Efficiency Analysis of one Class of Uniform Linear Phase FIR Filter Banks

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Abstract — **One class of uniform linear phase filter banks with different numbers of band-pass channels will be considered in this study, concentrating on 5, 9 and 17-band filter banks and their mutual comparison concerning delay and implementation complexity. Designed banks are based on the FIR filters and frequency response masking technique and are also compared to the banks with direct realization considering complementarity and delay.**

Keywords — **FIR filters, interpolation, uniform filter banks, FRM.**

I. INTRODUCTION

IN the light of technology breach that happened in the last decades and is currently reaching its maximum when we talk about processor speed and other properties limited by electronic components, a need for optimization of current solutions appeared, in order to satisfy growing needs for communications and society accustomed to constant technology improvement. Improvement has to be achieved at all levels, from electronic components and their implementation into the physical layer up to protocols and mechanisms used at higher layers.

When we talk about digital signal processors, they are a crucial component of a system. Their function of processing and delivering a signal determines a delay we shall have. Different techniques have been developed in order to overcome the limited processing speed of a processor.

Digital filter banks are frequently used in signal processing and realization of any communication system. Therefore, it's highly important for a filter bank to be as efficient as possible and not to require too much workload. A digital filter bank separates the signal to frequency sub-bands or composes the signal using two or more frequency sub-bands. This kind of approach enables parallel processing of different frequency bands of signal, thus contrives faster and more capacious processing which dislodges earlier restrictions – necessity of large memory elements and long delays.

Regarding appropriate filter type selection, FIR (*Finite Impulse Response*) filters are favorable due to their linear phase which makes them very popular to use [1]. However, a narrow transition region demands a higher filter order, which increases filter design complexity and processing time. Therefore, straightforward FIR filter design should be avoided. This can be managed using FRM (*Frequency Response Masking*) method that grants desirable properties without increasing complexity [2]. One possible realization of the linear phase uniform filter bank is presented in [1]. The filter bank is based on linear phase half-band FIR filters and FRM technique. Analysis of filter bank introduced in [1] is given in this study.

A brief description of multi-band uniform filter bank with a linear phase [1] is given in section II, results of efficiency analysis depending on sub-band number are shown in chapter III. Section IV presents concluding remarks.

II. MULTIBAND LINEAR PHASE FILTER BANK

Filter banks considered in this study are based on the idea of a uniform linear phase filter bank shown in [1]. Filter banks are designed using frequency response masking technique (FRM) [2]. FRM method is based on the interpolated [3] FIR filters. Observing the length N impulse response of a filter $h(z)$, interpolation by M, where M is an integer, imply inserting M-1 zeros between two taps of impulse response, where a new impulse response of interpolated filter $h_i(z)$ has $M(N\text{-}1)$+1 taps. For the transfer function of the prototype FIR linear phase filter of length N:

$$H(z)=\sum_{k=0}^{N-1}h(k)z^{-k} \tag{1}$$

where $h(k)$, k=0, 1, …, N-1 are filter coefficients, a corresponding transfer function of interpolated filter is:

$$H_i(z)=\sum_{k=0}^{N-1}h(k)z^{-kM} \tag{2}$$

where z is replaced with z^M. This results in frequency domain amplitude response compression by M times and leads to apparition of images on π/M, $2\pi/M$,..., π frequencies. Images are filtered in the second stage by a masking filter.

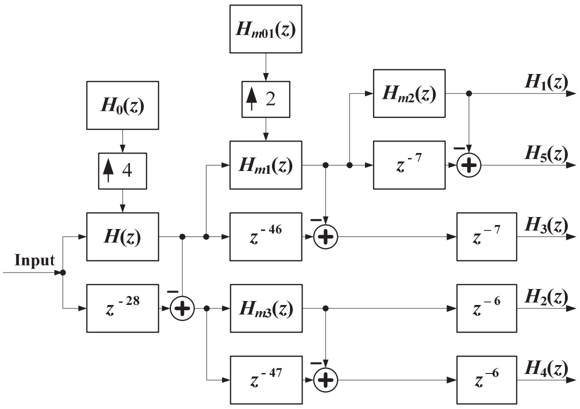

Fig. 1. Construction of a 5-band bank.

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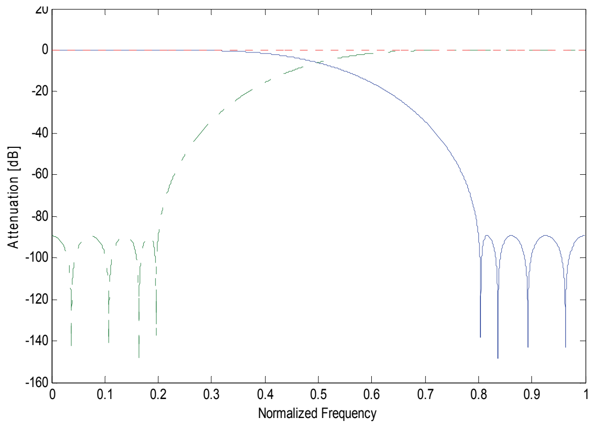

Fig. 2. Attenuation of prototype filter.
By interpolation of this filter and its complement, multi-band filters are derived, with bands that cover entire frequency range and are delay complementary.

A. *Filter bank design*

The principle of band separation used here was shown for the first time in [1], where a 9-band uniform bank is considered and instructions for its realization are given. Based on this idea, we have built uniform filter banks in this study. Similar design is used in [4], [5] for the realization of non-uniform filter banks for the hearing-aid devices.

Linear phase uniform filter bank design will be explained by a simple example of 5-channel filter bank. Realization of the 5-channel bank is shown in Fig. 1. Attenuation of a prototype half-band low-pass 14-th order filter $H_0(z)$ with a stop-band edge frequency of 0.8π and stop-band attenuation of 80dB and its complementary high-pass filter are shown in Fig. 2. Designed filters are delay complementary – [7], which is shown by a dashed line in Fig. 2. After interpolation b a factor M=4, filters $H(z)$, $H_{\mathrm{inv}}(z)$ are obtained, Fig. 3. A stop-band edge frequency of the first pass-band of the filter $H(z)$ is $0.8\pi/4=0.2\pi$, and *images* (additional pass-bands) appear at frequencies 0.5π and π. Filters $H(z)$ and $H_{\mathrm{inv}}(z)$ are also delay complementary.

Separation of previously shaped bands in order to form a bank is accomplished by masking filters [1], [2], [3]. Observe a filter with pass-band edge frequency f_p, and stop-band edge frequency f_s. According to this method, we will design a masking filter with pass-band edge frequency $f_{mp}=f_p$ and stop-band edge frequency $f_{ms}=\pi/M - f_s$, where $\pi/M - f_s$ is a lower stop-band limit of an *image* around frequency π/M. Considering example in Fig. 3 we will need a masking filter with parameters: $f_{mp1}=0.1\pi$, $f_{ms1}=0.3\pi$, $f_{mp2}=0.9\pi$, $f_{ms2}=0.7\pi$. It is built by interpolating a low-pass prototype masking filter H_{m01} by factor M=2, with stop-band limit $f_{m0s}=0.6\pi$ (Fig. 4).

Using this masking filter, bands 1 and 5 of the filter bank are realized. Band 3 is distinguished by using its complement. In order to separate bands 1 and 5, we need to construct a new masking filter - H_{m2}, with a stop-band edge frequency of 0.8π, and its complement (Fig. 5). Similarly, bands 2, and 4 (Fig. 6) are constructed by a third masking filter H_{m3} and its complementary filter H_{minv3}.

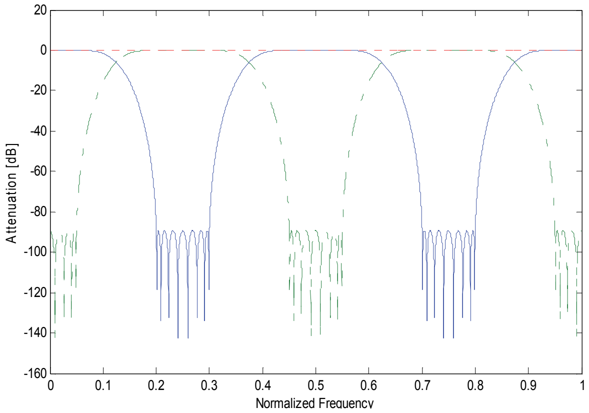

Fig. 3. Attenuation of interpolated filters, sub-bands of designed filter bank.

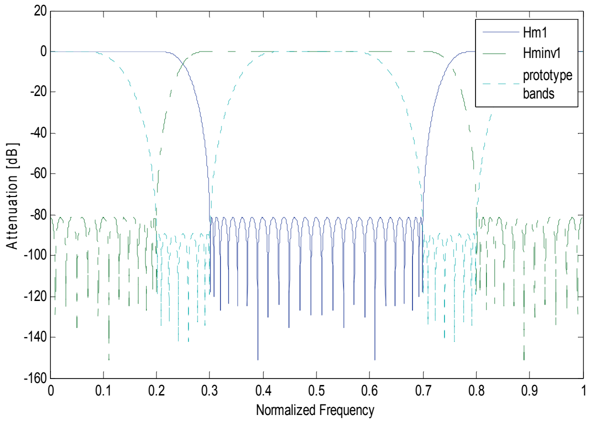

Fig. 4. First masking filter.

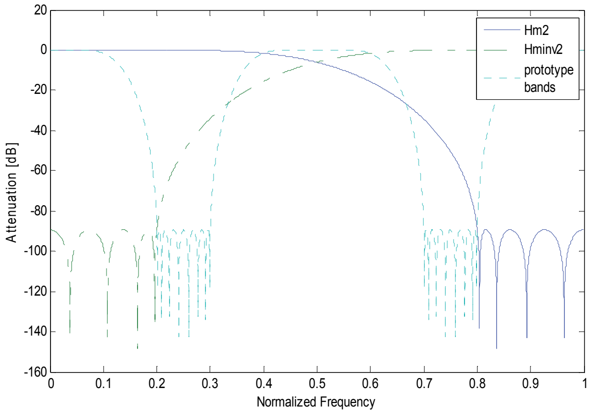

Fig. 5. Second masking filter.

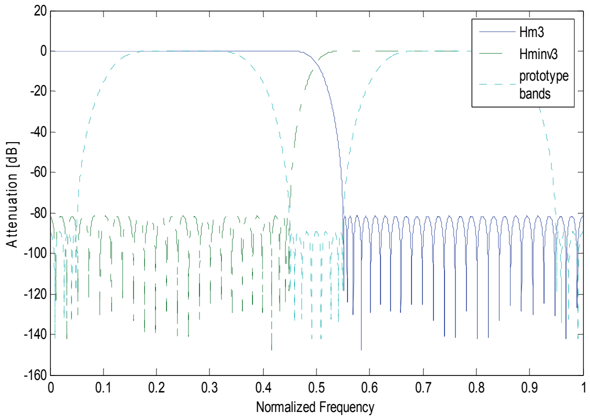

Fig. 6. Third masking filter.

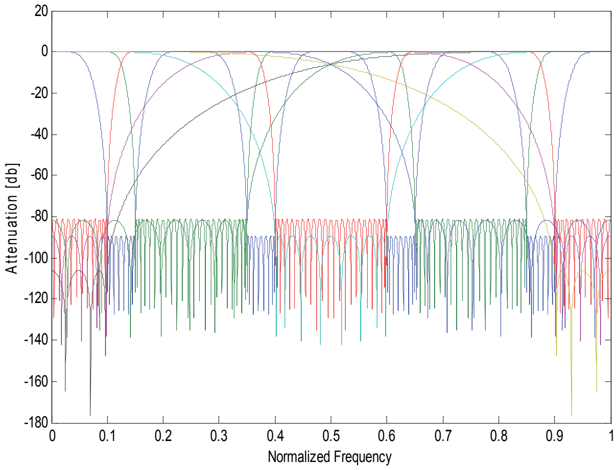

Fig. 7a) Masking filters 1-4 for a bank with 9 bands.

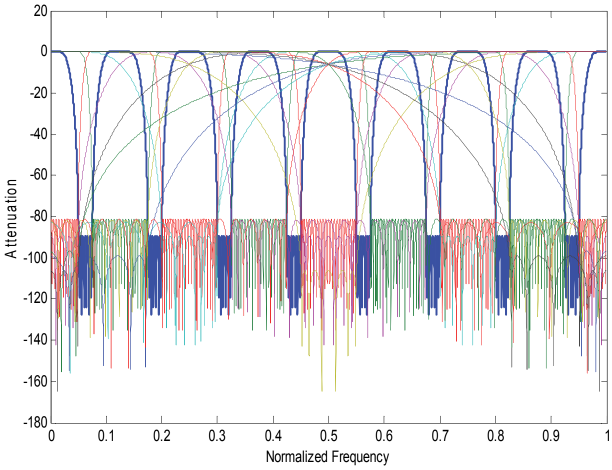

Fig. 7b) Masking filters 1-8 for a bank with 17 bands.

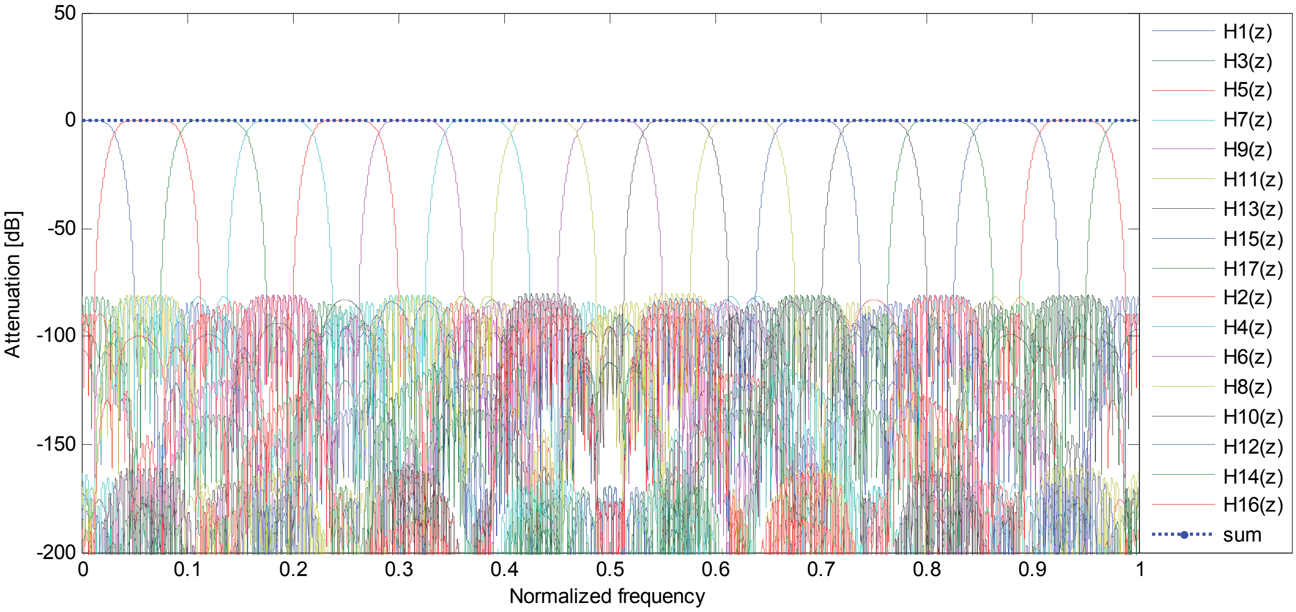

Fig. 7c) Attenuation of derived 17-band uniform filter bank.

B. *Phase correction*

Different bands of the proposed filter bank are obtained by a variable number of linear phase masking filters of different orders, i.e. each band has a different delay. If we combine these filters into an integral filter bank without phase correction, even if obtained amplitude behavior looks fine, distortions will appear in a processed signal due to unbalanced phase behavior of combined pass-band filters. Therefore, it is necessary to equalize the delays of all bands in order to avoid phase distortion of the processed signal. This is accomplished by extending the impulse response of each band to be equal to the length of the previously identified longest impulse response.

As previously mentioned, if a high order masking filter is used during extraction of any band, it will cause a long delay of the entire system. This is a problem we have to deal with in the realization of banks with a high number of sub-bands. As we shall see further in the text, filters with the highest filter order are those with pass-band edge frequencies near 0.5π. These should be dealt with in the interest of reducing a filter order to minimize the phase correction required.

C. *Banks with 9 and 17 bands*

All filters and their parameters required for realizing filter banks with 5, 9 and 17 bands are defined by Tables. In Tables 1, 3, 5 masking filters used for each bank and their stop-band frequencies, including a prototype filter and its parameters are shown. Some filters are constructed directly as half-band filters, while others are derived from prototype masking filters by interpolation - H_{m0x}, H_{mx} (prototype filter and interpolated filter, respectively). In the bottom of the table there is a stop-band limit of a prototype half-band filter (H_0) that gives, by interpolation, a group of bands that are further separated (H). In Tables 2, 4 and 6 transfer functions for all sub-bands are given. A subscript *inv* represents a complementary filter. Some of the masking filters used for 9-band and 17-band filter banks and attenuation of a derived uniform 17-band filter bank are shown in Figs 7a, 7b and 7c, respectively.

III. EFFICIENCY ANALYSIS

We will concentrate on three main properties of an efficient filter bank: processing delay, implementation complexity and complementarity. These are usually tightly

TABLE 1: MASKING FILTERS FOR A 5-BAND BANK.

Masking filter	*Stop-band frequency*	*Imp. resp. length (d_i)*	*No. mult. (m_i)*
H_{m01}, H_{m1}	0.6π, 0.3π	47, 93	24
H_{m2}	0.8π	15	8
H_{m3}	0.55π	95	48
H_o, H	0.8π, 0.2π	15, 57	8

TABLE 2: OUTPUTS OF A UNIFORM BANK WITH 5 BANDS.

Output	*Transfer function*	*D*
$H_1(z)$	$H_{m1}(z)H_{m2}(z)H(z)$	81
$H_2(z)$	$H_{m3}(z)H_{inv}(z)$	75
$H_3(z)$	$H_{minv1}(z)H(z)$	74
$H_4(z)$	$H_{minv3}(z)H_{inv}(z)$	75
$H_5(z)$	$H_{m1}(z)H_{minv2}(z)H(z)$	81

TABLE 3: MASKING FILTERS FOR A 9-BAND BANK.

Masking filter	*Stop-band frequency*	*Imp. resp. length (d_i)*	*No. mult. (m_i)*
H_{m01}, H_{m1}	0.6π, 0.15π	47, 185	24
H_{m02}, H_{m2}	0.8π, 0.4π	15, 29	8
H_{m3}	0.9π	11	6
H_{m4}	0.65π	31	16
H_{m05}, H_{m5}	0.55π, 0.275π	95, 189	48
H_{m6}	0.775π	15	8
H_{m7}	0.525π	187	94
H_o, H	0.8, 0.2π	15, 113	8

TABLE 4: OUTPUTS OF A UNIFORM BANK WITH 9 BANDS.

Output	*Transfer function*	*D*
$H_1(z)$	$H_{m1}(z)H_{m2}(z)H_{m3}(z)H(z)$	167
$H_2(z)$	$H_{m1}(z)H_{m4}(z)H(z)$	163
$H_3(z)$	$H_{m1}(z)H_{minv2}(z)H(z)$	162
$H_4(z)$	$H_{m1}(z)H_{m4}(z)H(z)$	163
$H_5(z)$	$H_{m1}(z)H_{m2}(z)H_{minv3}(z)H(z)$	167
$H_6(z)$	$H_{m5}(z)H_{m6}(z)H_{inv}(z)$	157
$H_7(z)$	$H_{m5}(z)H_{m7}(z)H_{inv}(z)$	243
$H_8(z)$	$H_{m5}(z)H_{minv7}(z)H_{inv}(z)$	243
$H_9(z)$	$H_{m5}(z)H_{minv6}(z)H_{inv}(z)$	157

interconnected. For example, good complementarity entails a longer delay and higher complexity, etc. An optimal solution should be found depending on the needs of a particular system.

A. Processing delay

There is a problem of time efficiency of these filter banks. As we can see, with an increasing number of bands, on one hand we need to use more masking filters, and on the other, better selectivity is required, i.e., a higher filter order. A high filter order is a disadvantage of interpolation – it extends the impulse response length of the filter applied to. This means that the delay time of a filter is multiplied by the previously mentioned factor of interpolation *M*.

All of these result in a longer delay during signal processing. The delay of each band is given by:

$$D = \frac{d-1}{2} + \sum_{i} \frac{d_i - 1}{2} \tag{3}$$

where d is the length of the impulse response of the interpolated prototype half-band filter (H), and d_i is the length of used masking filter(s) impulse response(s).

TABLE 5: MASKING FILTERS FOR A 17-BAND BANK.

Masking filter	*Stop-band frequency*	*Imp. resp. length (d_i)*	*No. mult. (m_i)*
H_{m01}, H_{m1}	0.6π, 0.075π	47, 369	24
H_{m02}, H_{m2}	0.8π, 0.2π	15, 57	8
H_{m03}, H_{m3}	0.9π, 0.45π	11, 21	6
H_{m4}	0.95π	7	4
H_{m5}	0.7π	23	12
H_{m06}, H_{m6}	0.65π, 0.325π	31, 61	16
H_{m7}	0.825π	15	8
H_{m8}	0.575π	63	32
H_{m09}, H_{m9}	0.525π, 0.2625	189, 373	95
H_{m010}, H_{m10}	0.55π, 0.1375π	95, 377	48
H_{m11}	0.8875π	11	6
H_{m12}	0.7625π	19	10
H_{m13}	0.6375π	35	18
H_{m14}	0.5125π	371	186
H_o, H	0.8π, 0.2π	15, 225	8

TABLE 6: OUTPUTS OF A UNIFORM BANK WITH 17 BANDS.

Output	*Transfer function*	*D*
$H_1(z)$	$H_{m1}(z)H_{m2}(z)H_{m3}(z)H_{m4}(z)H(z)$	337
$H_2(z)$	$H_{minv1}(z)H_{m6}(z)H_{m7}(z)H(z)$	333
$H_4(z)$	$H_{minv1}(z)H_{minv6}(z)H_{m8}(z)H(z)$	357
$H_5(z)$	$H_{m1}(z)H_{m2}(z)H_{minv3}(z)H(z)$	334
$H_6(z)$	$H_{minv1}(z)H_{minv6}(z)H_{minv8}(z)H(z)$	357
$H_7(z)$	$H_{m1}(z)H_{minv2}(z)H_{minv5}(z)H(z)$	335
$H_8(z)$	$H_{minv1}(z)H_{m6}(z)H_{minv7}(z)H(z)$	333
$H_9(z)$	$H_{m1}(z)H_{m2}(z)H_{m3}(z)H_{minv4}(z)H(z)$	337
$H_{10}(z)$	$H_{m9}(z)H_{m10}(z)H_{m11}(z)H_{inv}(z)$	491
$H_{11}(z)$	$H_{m9}(z)H_{minv10}(z)H_{m12}(z)H_{inv}(z)$	495
$H_{12}(z)$	$H_{minv9}(z)H_{minv10}(z)H_{m13}(z)H_{inv}$	503
$H_{13}(z)$	$H_{minv9}(z)H_{m10}(z)H_{m14}(z)H_{inv}(z)$	671
$H_{14}(z)$	$H_{minv9}(z)H_{m10}(z)H_{minv14}(z)H_{inv}(z)$	671
$H_{15}(z)$	$H_{minv9}(z)H_{minv10}(z)H_{minv13}(z)H_{inv}(z)$	503
$H_{16}(z)$	$H_{m9}(z)H_{minv10}(z)H_{minv12}(z)H_{inv}(z)$	495
$H_{17}(z)$	$H_{m9}(z)H_{m10}(z)H_{minv11}(z)H_{inv}$	491

Considering the 5-band bank, the longest delay appears during the separation of bands 1 and 5, and it is 81 taps (for a voice signal with a standard sampling frequency of 8 kHz, this is around 10 ms) – Table 2.

If we analyze the bank with 9 bands, we can see that the masking filter H_{m7} is significant because of its demand for a high selectivity which implies a long impulse response – Table 3. Therefore, bands 7 and 8 have the longest delay of 243 taps (around 30ms for a voice signal) – Table 4. There is a similar situation with the 17-band filter bank. The masking filter H_{m14} has the longest response (Table 5), thus bands 13 and 14 have the longest delay of 671 taps (around 80 ms for a voice signal) – Table 6.

These delays are sometimes longer than those achieved by direct realization. Beside a negative effect of interpolation, which is relevant for filter banks with an increasing number of bands, there is one more reason for the extended delay which is, actually, the most important. It is clear that in the construction of these banks the bands around the frequency of 0.5π are the most critical. In fact, for their extraction we need to build a masking filter with a stop-band frequency around 0.5π, and as these filters are half-band, they need to be extremely selective and therefore they will have a long impulse response which directly affects the entire system delay (Fig. 8). Increasing the number of bands, the stop-band frequency approaches 0.5π and we need more selective filters with an impulse response length rapidly rising along with processing delay.

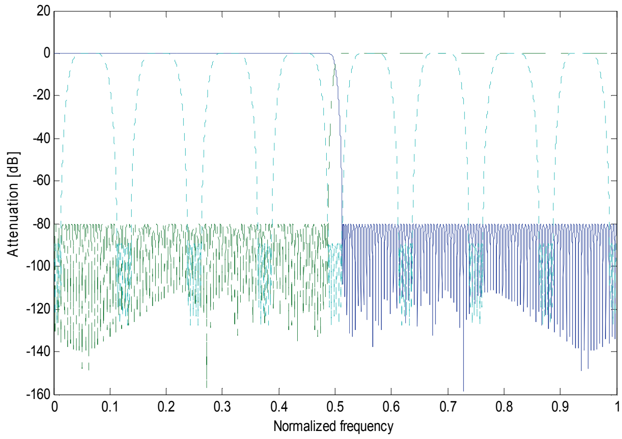

Fig. 8. Problem of a masking filter with high selectivity.

Overcoming this obstacle could be accomplished by using other functions for masking filter construction in MATLAB, in order to avoid the necessity of highly selective filters. Possible solutions are among the methods of optimization in [6].

B. Implementation complexity

The number of multiplications required for implementation of described filter banks is given by:

$$C=\sum_i m_i \qquad (4)$$

where $m_i=(d_i+1)/2$ is the number of multiplications needed for realization of all prototype FIR filters (Table 1, 3, 5).

In the previous Section we have seen the negative effects of interpolation on delay time. So why do we use it? Interpolation doesn't increase design complexity because zero-valued taps are inserted. Using it we can obtain very selective filters with a slight number of multiplications.

Also, complementary filters don't need separate realization - Fig. 1., they are simply obtained from their peer filters. As shown, these filters are delay complementary and if we use them in band separation the derived filter bank will save this property, opposite to a direct realization where such complementarity doesn't exist.

For a 5-band bank *C*=88 (Table 1). For a direct realization of a bank with the same characteristics we need C_{dir}=5·46=230 multiplications (5 bands with 46 multiplications for each sub-band). We can define

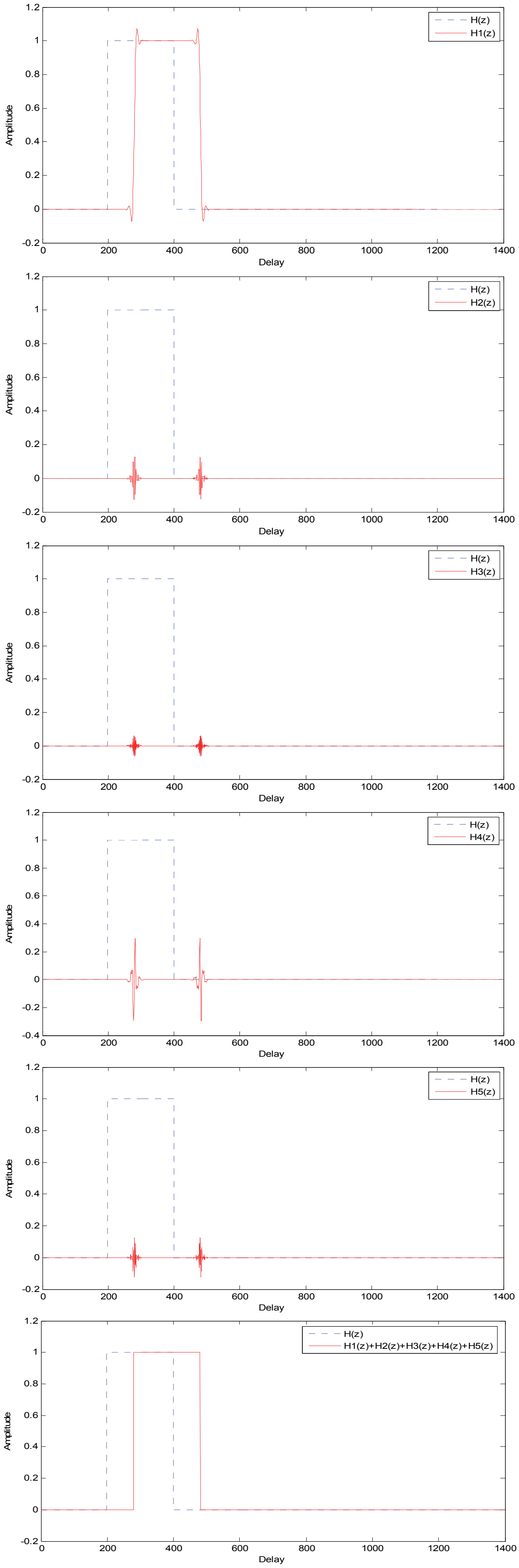

Fig. 9. Outputs of a 5-band filter bank.

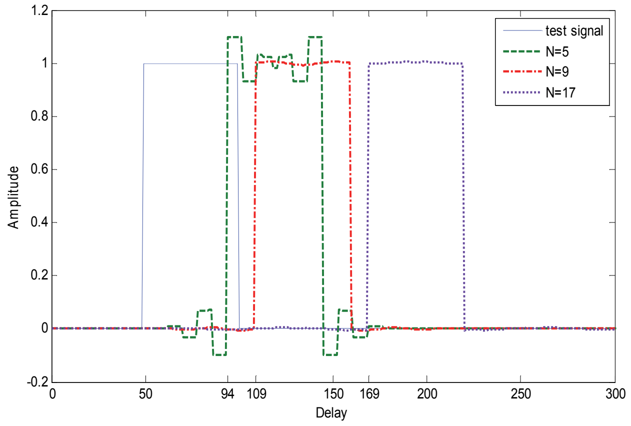

Fig. 10. Testing banks made by direct realization. Delays of 44 (N=5), 59 (N=9) and 119 (N=17) taps.

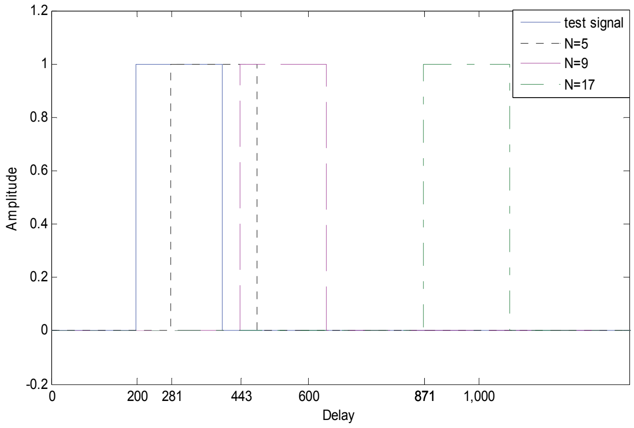

Fig. 11. Testing filter banks using rectangular impulse. Delays of 81 (N=5), 243 (N=9) and 671 (N=17) taps.

implementation reduction for our system as:

$$IR = \frac{C_{dir} - C}{C_{dir}} = 61.7\% \tag{5}$$

IR factor represents the savings in implementation resources that we can obtain by using this method for filter bank design compared to a direct realization. In Table 7 there is an overview of IR for all designed banks. As we can see, there is significant implementation reduction for all tested filter banks. In the case of a 17-band filter bank there are approximately 76.6% of savings which means direct realization requires 4 times the number of multiplications needed with FRM technique.

TABLE 7: NUMBER OF MULTIPLICATIONS COMPARISON.

	C_{dir}	C	IR
$N = 5$	230	88	61.7 %
$N = 9$	549	212	61.4 %
$N = 17$	2057	481	76.6 %

C. Complementarity

Complementarity is the most important property of a filter bank. While the other two mentioned can be variable due to system requirements, this one is not negotiable. If our filter bank is not delay complementary, it will cause distortions to a processed signal and in some cases make it inapplicable to further analysis.

All bands of designed filter banks are delay complementary. Use of FRM technique and interpolated filters allows this. Bands of directly realized banks are not complementary because of separate and unadjusted realization of each band in a filter bank (Fig. 10), which favors use of FRM method though it has longer delays. In Fig.10 we can see distortions to a rectangular signal that happened after processing it with a filter bank.

Banks that we have analyzed in this study are tested using rectangular signal brought to the input of the system. Outputs of all bands are summed and derived assembled signal for all filter banks is shown (Fig. 9, 11) along with the testing impulse. It is clear that neither of the banks brings any of phase or amplitude distortions, while with increasing number of bands there is longer delay.

IV. CONCLUSION

As we can see, used techniques for filter banks realization result with significant implementation reduction comparing to direct realization. Property of delay complementarity of these banks makes them more suitable than direct realization that cannot provide satisfactory complementarity. However, with an increasing number of bands in filter banks, delays become significant and further analysis of these banks should be focused on overcoming this problem.

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