Performance Analysis of a Cooperative Diversity Scheme in Rician Fading Channel

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Abstract — **In this paper we consider performance of a cooperative diversity scheme in Rician fading channel. The analytical results obtained in this paper will be comapred to the simulation results published previously, and it will be shown that the results are in great accordance. Besides that, the influence of the number of relay nodes on the performance of the system will be analysed.**

Keywords — **Fading channel, network cooperative diversity, outage probability, wireless networks.**

I. INTRODUCTION

During the past few years a special attention is paid to cooperative diversity protocols [1], [2], [3]. Such a protocols may significantly improve the performance of wireless communication systems. Using this protocols it is possible to create additional paths between the source and destination using other users' terminals which serve as intermediate relay nodes. For example, in [2] the authors proposed a algorithm which enables the source and the relay to adjust phases of their transmission so that the two signals may be combined coherently at the destination. The problem is that the algorithm requires prior knowledge of the channel. Besides that, the beamforming requires significant modifications of the radio frequency front-end that increase complexity and cost. In all the mentioned papers, it was assumed that there is one source, one relay, and one destination. On the other side, papers [4] and [5] considered the case of multiple relays. Paper [4] studied the case of orthogonal transmission between the source and relay. In [5], this orthogonality constraint was relaxed, the source and relay are allowed to transmit simultaneosly, and it was shown that in this way a significant performance improvement could be achieved at the cost of higher complexity decoder. Unfortunately, all of these considerations were mostly theoretic, and practical implementation was not considered. There are a number of difficulties in practical implementation of this system compared to fixed MIMO systems. For example, the number of antennas in such a system is not fixed and it depends on the number of relays being used for the communication. Also, the channel state information is not known in advance to the relays and it needs to be transmitted over the imperfect channel.

In [6] a simple novel scheme that selects the best relay between source and destination based on instantaneous channel measurements is proposed. The proposed scheme requires no knowledge of the topology or its estimation. The technique is based on signal strength measurements rather than distance and requires a small fraction of the channel coherence time. Additionally, the algorithm itself provides for the necessary coordination in time and group formation among the cooperating terminals. Also, the simplicity of the technique allows immediate implementation in existing radio hardware. Paper [6] considers the probability of collision, the probabilty that two or more relays transmit the information from source to destination, in Rayleigh and Rician channel. For Rayleigh channel there are both simulation and theoretical results, but there are only simulation results for the Rician channel case. Therefore, in this paper we give a theoretical analysis of probability of collision in Rician channel.

In the following sections we firt give a brief description of the protocol and results from [6], and the an analysis of collision probability in Rician fading channel is given.

II. SYSTEM MODEL

According to the protocol proposed in [6], a single relay among a set of M relay nodes is selected, depending on which relay provides for the “best” end-to-end path between source and destination (Figs. 1 and 2 in [6]). The wireless channel a_{si} between source and each relay i, as well as the channel a_{id} between relay i and destination affect performance. These parameters model the propagation environment between any communicating terminals and change over time, with a rate that macroscopically can be modeled as the Doppler shift, inversely proportional to the channel coherence time. Opportunistic selection of the “best” available relay involves the discovery of the most appropriate relay, in a distributed and “quick” fashion, well before the channel changes again. In that way, topology information at the relays (specifically location coordinates of source and destination at each relay) is not needed.

More specifically, the relays overhear a single

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transmission of a ready-to-send (RTS) packet and a clear-to-send (CTS) packet from the destination. From these packets, the relays assess how appropriate each of them is for information relaying. The transmission of RTS from the source allows for the estimation of the instantaneous wireless channel a_{si} between source and relay i, at each relay i. Similarly, the transmission of CTS from the destination allows for the estimation of the instantaneous wireless channel a_{id} between relay i and destination at each relay i.

Since communication among all relays should be minimized for reduced overall overhead, a method based on time was selected: as soon as each relay receives the CTS packet, it starts a timer from a parameter h_i based on the instantaneous channel measurements a_{si}, a_{id}. The timer of the relay with the best end-to-end channel conditions will expire first. That relay transmits a short duration flag packet, signaling its presence. All relays, while waiting for their timer to reduce to zero (i.e., to expire), are in listening mode. As soon as they hear another relay to flag its presence or forward information (the best relay), they back off.

The channel estimates a_{si}, a_{id} at each relay, describe the quality of the wireless path between source-relay-destination, for each relay i. Since the two hops are both important for end-to-end performance, each relay should quantify its appropriateness as an active relay, using a function that involves the link quality of both hops. Two functions are used in [6]: under Policy I, the minimum of the two is selected, while under Policy II, the harmonic mean of the two is used. Policy I selects the "bottleneck" of the two paths while Policy II balances the two link strengths and it is a smoother version of the first one. It was shown in [6] that the Policy I has better performances, and it will be used in this paper. Under Policy I

$$h_i = \min\{|a_{si}|^2, |a_{id}|^2\} \tag{1}$$

The relay that maximizes function h_i is the one with the "best" end-to-end path between initial source and final destination. After receiving the CTS packet, each relay i will start its own timer with an initial value T_i, inversely proportional to the end-to-end channel quality h_i, according to the following equation:

$$T_i = \frac{\lambda}{h_i}, \tag{2}$$

where λ is a constant and has the units of time.

III. PERFORMANCE ANALYSIS

The probability of having two or more relay timers expire "at the same time" is zero. However, the probability of having two or more relay timers expire within the same time interval c, and therefore cause a collision, is nonzero and was evaluated in [6]. It was shown that the probability of collision is equal to

$$P_c = 1 - M(M-1)\int_c^{\infty} f(y)[1-F(y)]^{M-2} F(y-c)dy \tag{3}$$

where $f(x)$ is the probability density function (pdf), and $F(x)$ is the cumulative distribution function (cdf) of random variables $T_i, i = 1,...,M$. The cumulative distribution function and probability density function are related to respective distributions of h_i in the following way:

$$F(t) \equiv \text{cdf}_{T_i}(t) = 1 - \text{cdf}_{h_i}(\lambda / t) \tag{4}$$

$$f(t) \equiv \text{pdf}_{T_i}(t) = \frac{d}{dt}F(t) = \frac{\lambda}{t^2}\text{pdf}_{h_i}(\lambda / t) \tag{5}$$

In case of Rayleigh channel, it was shown in [6] that

$$F(t) = \exp\left(-(\beta_1 + \beta_2)\frac{\lambda}{t}\right) \tag{6}$$

$$f(t) = \frac{\lambda(\beta_1 + \beta_2)}{t^2}\exp\left(-(\beta_1 + \beta_2)\frac{\lambda}{t}\right) \tag{7}$$

where β_1 and β_2 are parameters of the exponential random variables $|a_{si}|^2$ and $|a_{id}|^2$.

In case of more general of Rician channel, it will be assumed that $|a_{si}|$ and $|a_{id}|$, $i = 1,...,M$, are independent (but not identically distributed) Rician random variables with the following pdfs, respectively:

$$\begin{aligned} p_1(x) &= \frac{x}{\beta_1^2}\exp\left(-\frac{x^2+\alpha_1^2}{2\beta_1^2}\right)I_0\left(\frac{x\alpha_1}{\beta_1^2}\right) \\ p_2(x) &= \frac{x}{\beta_2^2}\exp\left(-\frac{x^2+\alpha_2^2}{2\beta_2^2}\right)I_0\left(\frac{x\alpha_2}{\beta_2^2}\right) \end{aligned} \tag{8}$$

The probability density functions of $|a_{si}|^2$ and $|a_{id}|^2$ are given by a noncentral chi-squared distribution:

$$\begin{aligned} p_{1sq}(x) &= \frac{1}{2\beta_1^2}\exp\left(-\frac{x+\alpha_1^2}{2\beta_1^2}\right)I_0\left(\frac{\sqrt{x}\alpha_1}{\beta_1^2}\right) \\ p_{2sq}(x) &= \frac{1}{\beta_2^2}\exp\left(-\frac{x+\alpha_2^2}{2\beta_2^2}\right)I_0\left(\frac{\sqrt{x}\alpha_2}{\beta_2^2}\right) \end{aligned} \tag{9}$$

The cumulative distribution functions of $|a_{si}|^2$ and $|a_{id}|^2$ are [7]:

$$P_{ksq}(x) = 1 - Q_1\left(\frac{\alpha_k}{\beta_k}, \frac{\sqrt{x}}{\beta_k}\right), \quad k = 1, 2 \tag{10}$$

where $Q_1(a,b)$ is the Marcum function defined by

$$Q_1(a,b) = \exp\left(-\frac{a^2+b^2}{2}\right)\sum_{k=0}^{\infty}\left(\frac{a}{b}\right)^k I_k(a \cdot b) \tag{11}$$

The pdf of the minimum of random variables $|a_{si}|^2$ and $|a_{id}|^2$ is [8]:

$$\begin{aligned} p_{\min}(x) &= p_{1sq}(x)(1-P_{2sq}(x)) + p_{2sq}(x)(1-P_{1sq}(x)) \\ &= p_{1sq}(x)Q_1\left(\frac{\alpha_1}{\beta_1}, \frac{\sqrt{x}}{\beta_1}\right) + p_{2sq}(x)Q_1\left(\frac{\alpha_2}{\beta_2}, \frac{\sqrt{x}}{\beta_2}\right) \end{aligned} \tag{12}$$

Finally,

$$f(t) = \frac{\lambda}{t^2}p_{\min}\left(\frac{\lambda}{t}\right) \tag{13}$$

$$F(t) = 1 - \int_0^x p_{\min}(u)du \tag{14}$$

IV. NUMERICAL RESULTS

Fig. 1. shows the probability density function (12) and the same pdf obtained by Monte-Carlo simulation. It is clear that there is an excellent match between the analytical result in (12) and the Monte Carlo simulation.

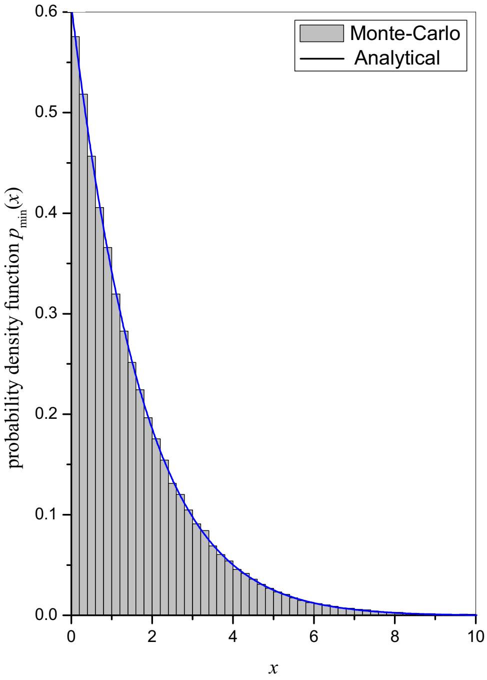

Fig. 1. Comparison between the analytical results for the pdf (12) and the Monte Carlo simulation ($\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1$, and 200000 iterations).

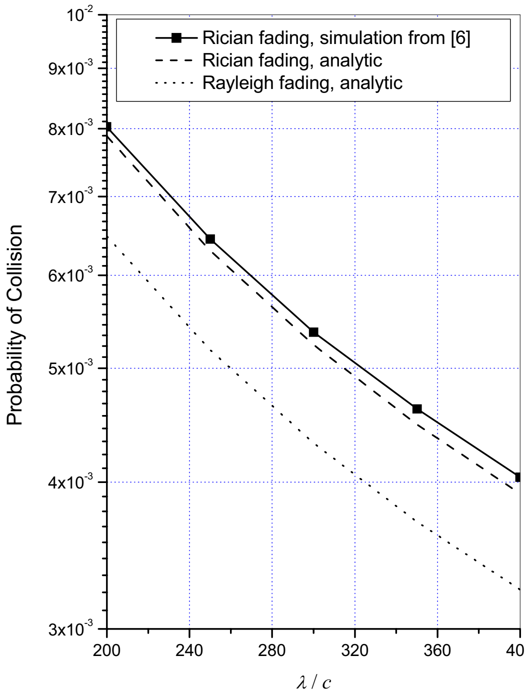

Fig. 2. Probability of collision as a function of λ / c

The probability of collision as a function of λ / c ratio is shown in Fig. 2. The number of relay stations is $M = 6$. The parameters are $\beta_1 = \beta_2 = 1$ for Rayleigh fading. In case of Rician fading, it was chosen that both direct and diffuse component have the same power, and the total signal power is equal to 1. It can be seen that anlytical results from this paper are close to Monte-Carlo simulation results from [6].

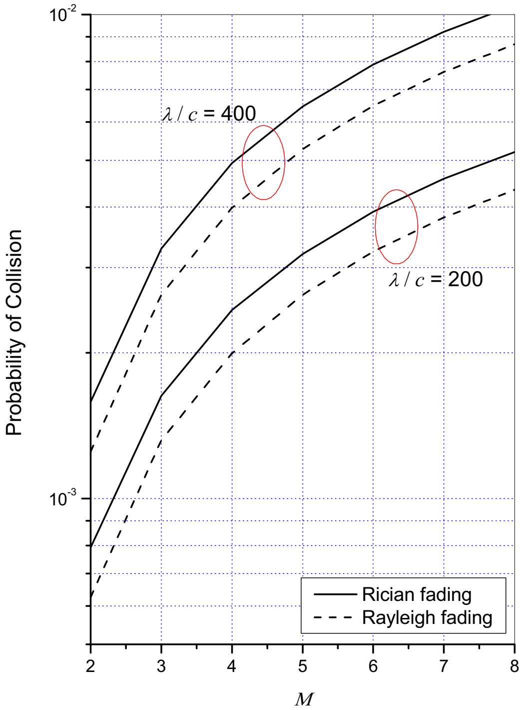

Fig. 3. Probability of collision as a function of the number of relay nodes M

Fig. 3 shows the probability of collision as a function of the number of relay nodes. As expected, it can be seen that the higher the number of relay nodes, the higher the error probability.

V. CONCLUSION

An analysis of a cooperative diversity system in the presence of Rician fading is considered in this paper. It was shown that the analytical results are in great accordance with the ones obtained by Monte-Carlo simulation. Also, the results show that the probability of collision increases with the number of realy nodes.

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