Two-Channel IIR Filter Banks Utilizing the Frequency-Response Masking Technique*

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Abstract **— In this paper, a new approach for a two-channel IIR filter bank based on the frequency-response masking technique is presented. A model filter pair is a double complementary IIR filter pair implemented as a parallel connection of two all-pass filters. Masking filters are linear-phase FIR filters. The resulting overall filter pair is nearly power complementary, and simultaneously achieves high sub-channel selectivity. The approximately linear phase of channel filters is achieved with an IIR model filter pair of an approximately linear phase. Compared to the traditional solution based on FIR filters only, the proposed filter bank exhibits a smaller overall delay and requires a smaller number of multipliers for implementation.**

Key words **— complementary filter pairs, digital filters, filter banks, frequency-response masking.**

I. Introduction

THE frequency-response masking (FRM) technique suitable for constructing digital filters with very narrow transition bandwidths was first introduced by Lim [1]. Utilizing the FRM approach, a single filter of a very high order can be replaced with several sub-filters of a considerably lower order thus reducing the overall computational complexity. The application of FRM based filters contributes to the spectral efficiency since the high-performance digital filters with sharp pass/stop-band transition can be achieved [1] – [4].

During the last decade, the applications of FRM technique in constructing two-channel filter banks with narrow transition bands have been considered [5], [6]. Two-channel filter banks are used to split the input signal in two adjacent sub-bands for some signal processing purposes, and to combine the two signal components in order to synthesize a desired composite signal. Two-channel filter banks are widely used building blocks in many applications such as coding, scrambling, de-noising, speech and music processing, image processing, and others. Moreover, two channel filter banks are used for constructing multichannel multilevel filter banks.

Various solutions for two-channel filter banks have been developed, and the choice of the "best" solution is closely related with the application in hand. A particular problem, frequently met in practice, is to construct a filter bank which achieves simultaneously high sub-band selectivity and a nearly perfect reconstruction property. This is of importance for processing the signals whose parameters change with time. Recently, the frequency-response masking technique has been introduced to synthesize two-channel filter banks with the major goal to lower the transition region between the low-pass and high-pass sub-channels [4], [5]. With the FRM approach the channel selectivity is increased, and the spectral efficiency of sub-channels is improved. Furthermore, the overall computational complexity is reduced as compared with the classical two-channel filter banks.

The synthesis of FIR two-channel maximally decimated filter bank based on the frequency-response masking (FRM) technique has been introduced in [4]. The FRM filter bank from [4] is a low-pass/high-pass filter bank with equal sub-channel bandwidths, and the overall bank satisfies a nearly perfect reconstruction property. An approach for the synthesis of FRM-based two-channel filter banks with unequal sub-channel bandwidths suitable for the rational sampling rate conversions has been published in [5].

The purpose of this paper is to introduce IIR filters in the structure of FRM based two-channel filter banks with the goal to improve the computational efficiency and to lower the overall delay of the bank. We concentrate on the maximally decimated filter banks with equal sub-channel bandwidths. We use a complementary IIR filter pair as a periodic model filter pair, and linear-phase FIR filters as masking filters. The efficiency of the realization is demonstrated through examples, and the properties of the implemented filter bank are discussed.

II. Maximally decimated two-channel filter bank

The block diagram of the two-channel maximally decimated filter bank with the processing unit between the analysis and synthesis part is shown in Fig. 1(a). The analysis bank consists of the low-pass and high-pass filters $H_{a0}(z)$ and $H_{a1}(z)$ followed by the factor-of-two down-samplers. The synthesis bank contains the factor-of-two up-samplers and the low-pass and high-pass filters $H_{s0}(z)$ and $H_{s1}(z)$. The crossover frequency ω_c is located at the centre of the baseband, i.e. at the angular frequency $\omega = \pi/2$ as indicated in Figure 1(b). The distances of the pass-band and stop-band edges from the crossover frequency are the same. The input-output relation of the system of

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Figure 1(a) when the processing unit is omitted is expressed as [4], [6],

$$Y(z) = T_0(z)X(z) + T_1(z)X(-z) \tag{1}$$

where $T_0(z)$ denotes the distortion transfer function, and the effects of aliasing produced by the down-sampling operations are represented with $T_1(z)$. Therefore, $T_1(z)$ is referred to as the aliasing transfer function. The filter bank is said to be a perfect reconstruction filter bank if $T_0(z) = z^{-K}$, K is a positive integer, and $T_1(z) = 0$. In that case, the signal at the output $y[n]$ is a delayed copy of the input $x[n]$, i.e., $y[n] = x[n - K]$. In many practical applications, the requirements for perfect reconstruction can be relaxed, and the nearly perfect reconstruction filter banks providing $y[n] \approx x[n - K]$, with $T_0(z) \approx z^{-K}$ and $T_1(z) \approx 0$, can be used.

Functions $T_0(z)$ and $T_1(z)$ are determined by the analysis and synthesis filters,

$$T_0(z) = H_{a0}(z)H_{s0}(z) + H_{a1}(z)H_{s1}(z), \tag{2}$$

$$T_1(z) = H_{a0}(-z)H_{s0}(z) + H_{a1}(-z)H_{s1}(z). \tag{3}$$

In quadrature mirror (QMF) filter banks, analysis pairs $[H_{a0}(z)\ H_{a1}(z)]$ and synthesis pairs $[H_{s0}(z)\ H_{s1}(z)]$ satisfy the power complementary property, and relations between the filters $H_{a0}(z,)$ $H_{a1}(z)$, $H_{s0}(z)$, $H_{s1}(z)$ are chosen in such a manner that the aliasing produced in the analysis bank is cancelled in the synthesis bank [6, p. 805].

III. FRM-BASED TWO-CHANNEL FILTER BANK

The frequency-response masking approach introduced in [4] enables one to generate a nearly perfect reconstruction filter bank with a narrow transition band between the channels. With the aid of FRM approach, analysis filters $H_{a0}(z)$, $H_{a1}(z)$, and synthesis filters $H_{s0}(z)$, $H_{s1}(z)$ are generated by combining a periodic model filter pair $[G(z^L)\ G_c(z^L)]$ with the appropriate set of masking filters. The filtering scheme is similar to the basic FRM algorithm introduced in [1]. As given in [4], the filter transfer functions in the two-channel filter bank of Fig. 1 are generated in the following manner

$$H_{a0}(z) = G(z^L)F_0(z) + G_c(z^L)F_1(z) \tag{4}$$

$$H_{a1}(z) = G(z^L)E_0(z) + G_c(z^L)E_1(z) \tag{5}$$

$$H_{s0}(z) = G(z^L)F_0(z) - G_c(z^L)F_1(z) \tag{6}$$

$$H_{s1}(z) = G(z^L)E_0(z) - G_c(z^L)E_1(z) \tag{7}$$

where $F_0(z)$ and $F_1(z)$ are low-pass, and $E_0(z)$ and $E_1(z)$ high-pass masking filters. Notice that $F_0(z)$, $F_1(z)$, $E_0(z)$, $E_1(z)$ are even-order linear-phase FIR filters.

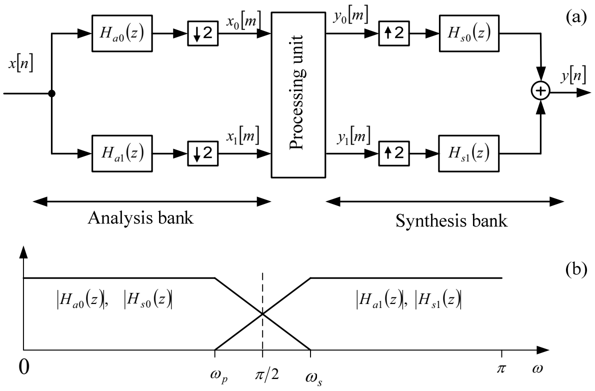

Fig. 1 Two-channel maximally decimated filter bank: (a) Block diagram. (b) Channel division.

Fig. 2 illustrates one of the two filtering schemes [4] for the analysis filters $H_{a0}(z)$ and $H_{a1}(z)]$ that is used in the following sections of this paper.

Fig. 2(a) shows the magnitude response of the model filter pair $[G(z)\ G_c(z)]$, whereas the periodic model filter pair $[G(z^L)\ G_c(z^L)]$ is shown in Fig. 2(b). Here, L should be an odd integer to enable the magnitude symmetry of $[H_{a0}(z)\ H_{a1}(z)]$ in respect to the centre of the baseband $\omega = \pi/2$. The magnitude responses of masking filters $F_0(z)$, $F_1(z)$, $E_0(z)$ and $E_1(z)$ are shown in Figure 2(c). Fig. 2(d) illustrates the resulting filter pair $[H_{a0}(z)\ H_{a1}(z)]$.

It was shown in [2], [4] that depending on the relations between the masking filters, several types of filter banks can be obtained. We use the following combination

$$F_1(z) = -\left(z^{-N_F/2} - (-1)^{N_F/2} F_0(-z)\right) \tag{8}$$

$$E_0(z) = F_1(-z) \tag{9}$$

$$E_1(z) = F_0(-z) \tag{10}$$

where N_F, the order of $F_0(z)$, is an even number. Notice that $F_1(z)$, $E_0(z)$ and $E_1(z)$ are completely determined by $F_0(z)$. The selected combination provides an efficient realization structure as will be shown later on in this paper.

The pass-band and the stop-band edge frequencies of the model pair $[G(z)\ G_c(z)]$, $\omega_p^{(G)}$ and $\omega_s^{(G)}$, satisfy the symmetry condition $\omega_s^{(G)} = \pi - \omega_p^{(G)}$, and the transition bandwidth is $(\omega_s^{(G)} - \omega_p^{(G)})$. In the periodic model pair $[G(z^L)\ G_c(z^L)]$, the transition bandwidth is L times smaller and amounts to $(\omega_s^{(G)} - \omega_p^{(G)})/L$. With an appropriate choice of masking filters the resulting overall filter pair $[H_{a0}(z)\ H_{a1}(z)]$ has the transition bandwidth of the periodic model filters as indicated in Fig. 2. The pass-band/stop-band edge frequencies of $[H_{a0}(z)\ H_{a1}(z)]$, $\omega_p^{(H)}$ and $\omega_s^{(H)}$ are determined as follows

$$\begin{aligned} \omega_p^{(H)} &= \pi/2 - (\omega_s^{(G)} - \omega_p^{(G)})/2L, \\ \omega_s^{(H)} &= \pi/2 + (\omega_s^{(G)} - \omega_p^{(G)})/2L. \end{aligned} \tag{11}$$

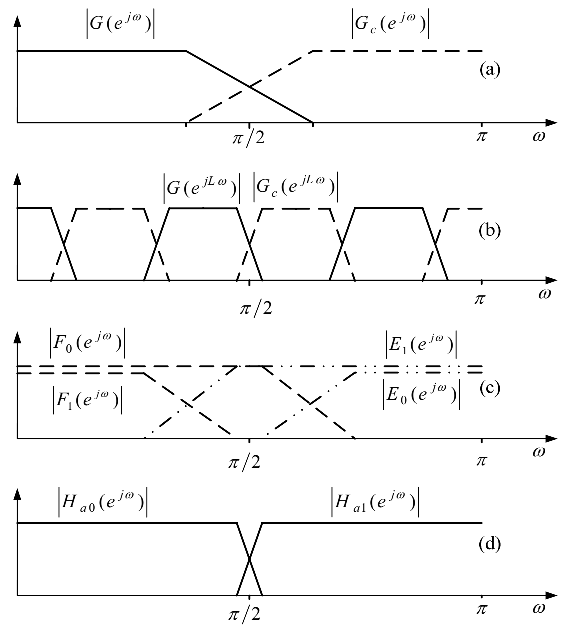

Fig. 2. Generating the analysis filter bank with the aid of FRM approach.

The edge frequencies of masking filter $F_0(z)$ are determined by expressions [4],

$$\omega_p^{(F_0)} = \left(2k\pi + \omega_s^{(G)}\right)/L\,,\ \omega_s^{(F_0)} = \left(2(k+1)\pi - \omega_s^{(G)}\right)/L, \quad (12)$$

where k is an integer. For the choice of k see [1] and [4].

In the filter banks presented in [4], the model filter pairs $[G(z)\ G_c(z)]$ are half-band FIR filter pairs that satisfy QMF bank properties [6]. In this paper, we consider FRM-based filter banks where the model filter pair $[G(z)\ G_c(z)]$ is an IIR half-band filter pair implemented as a parallel combination of two all-pass filters $A_0(z)$ and $A_1(z)$. Thus, we use

$$G(z) = \left(A_0\left(z^2\right) + z^{-1}A_1\left(z^2\right)\right)/2 \quad (13)$$

$$G_c(z) = \left(A_0\left(z^2\right) - z^{-1}A_1\left(z^2\right)\right)/2\,. \quad (14)$$

The filter pair $[G(z)\ G_c(z)]$ satisfies the power-complementary and all-pass complementary properties and can be used to construct IIR QMF banks [6].

IV. Efficient Realization Structure

In this section, we demonstrate how an efficient realization structure can be achieved for the analysis bank $[H_{a0}(z)\ H_{a1}(z)]$. The basic idea is to utilize equations (4), (5), (8)–(10) to represent the transfer functions $H_{a0}(z)$ and $H_{a1}(z)$ as a combination of functions $Q_0(z^2)$ and $z^{-1}Q_1(z^2)$ in order to obtain the following form:

$$H_{a0}(z) = Q_0\left(z^2\right) + z^{-1}Q_1\left(z^2\right) \quad (15)$$

$$H_{a1}(z) = Q_0\left(z^2\right) - z^{-1}Q_1\left(z^2\right) \quad (16)$$

This form enables one to represent $H_{a0}(z)$ and $H_{a1}(z)$ from Fig. 1 as the efficient polyphase structure shown in Fig. 3.

In the next step, we express the polyphase functions $Q_0(z^2)$ and $Q_1(z^2)$ with the aid of the polyphase components of model and masking filters. For model filters $[G(z),\ G_c(z)]$, the all-pass functions $A_0(z)$ and $A_1(z)$ from (13) and (14) represent the polyphase components. For masking filters, we utilize relations (8)–(10) to express the masking filters F_0(z), F_1(z), E_0(z), E_1(z) in terms of the polyphase components that correspond to F_0(z). Consequently, when representing F_0(z) in terms of the polyphase components P_0(z) and P_1(z),

$$F_0(z) = P_0\left(z^2\right) + z^{-1}P_1\left(z^2\right), \quad (17)$$

the remaining masking filters are expressible as follows

$$F_1(z) = -z^{-N_F/2} + P_0\left(z^2\right) - z^{-1}P_1\left(z^2\right), \quad (18)$$

$$E_0(z) = -z^{-N_F/2} + P_0\left(z^2\right) + z^{-1}P_1\left(z^2\right), \quad (19)$$

$$E_0(z) = P_0\left(z^2\right) - z^{-1}P_1\left(z^2\right), \quad (20)$$

where we assume that $N_F/2$ is even. Similar expressions are obtained for $N_F/2$ odd.

Finally, we take equations (4) and (5) and insert the substitutions from (13), (14), (17)–(20) and obtain the following expressions for $H_{a0}(z)$ and $H_{a1}(z)$

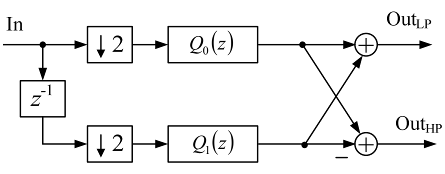

Fig. 3. Polyphase realization of the analysis bank.

$$H_{a0}(z) = A_0\left(z^{2L}\right)P_{0m}\left(z^2\right) + z^{-(L+1)}A_1\left(z^{2L}\right)P_1\left(z^2\right) + z^{-1}\left[0.5z^{-(L-1)}z^{-\left(\frac{N_F}{2}\right)}A_1\left(z^{2L}\right)\right] \quad (21)$$

$$H_{a1}(z) = A_0\left(z^{2L}\right)P_{0m}\left(z^2\right) + z^{-(L+1)}A_1\left(z^{2L}\right)P_1\left(z^2\right) - z^{-1}\left[0.5z^{-(L-1)}z^{-\left(\frac{N_F}{2}\right)}A_1\left(z^{2L}\right)\right] \quad (22)$$

where P_{0m}(z) represents the modified polyphase component P_0 (z) defined by

$$P_{0m}\left(z^2\right) = P_0\left(z^2\right) - 0.5z^{-N_F/2}\,. \quad (23)$$

Let us compare the expressions (21), (22) with the general polyphase forms as given in (15), (16). One observes that the first rows in equations (21) and (22) are identical and that they correspond to the branch Q_0(z). In the same manner, one concludes that the expressions in the brackets of the second rows correspond to Q_1(z). According to the above, we develop the realization structure of the analysis bank $[H_{a0}(z)\ H_{a1}(z)]$ as depicted in Fig. 4.

The resulting structure contains only two polyphase components of the masking FIR filter $F_0(z)$, three all-pass filters and delay elements. The arithmetic operations in the structure are evaluated at the rate of the output signal. The efficient realization structure for the synthesis bank can be evaluated in a similar manner.

V. Illustrative Example

The frequency response of the model filter pair $[G(z)\ G_c(z)]$ should satisfy the all-pass complementary and power-complementary properties [1], [4], i.e.,

$$\left|G\left(e^{j\omega}\right) + G_c\left(e^{j\omega}\right)\right| = 1\,,\ \left|G\left(e^{j\omega}\right)\right|^2 + \left|G_c\left(e^{j\omega}\right)\right|^2 = 1 \quad (24)$$

The conditions stated above can be met with an IIR half-band filter whose transfer function is expressible as a combination of two all-pass filters according to (13) and (14). When the phase nonlinearity can be tolerated, an elliptic half-band filter [7] is a suitable choice. A nearly linear phase can be achieved with the approximate linear phase IIR half-band filters [8]. The realization structure of the analysis bank shown in Fig. 4 is an efficient solution for both types of model filter pairs. For the design of linear-phase masking filter $F_0(z)$, the Rabiner, McClellan, Parks algorithm [9] can be used. The remaining masking filters are generated from the relations given in (8)–(10).

Figs. 5–8 show the results obtained for a two-channel FRM filter bank composed of the 10[th] order IIR half-band model filter of approximately linear phase, and the 36[th] order linear-phase FIR masking filters, with k =1 and L=5.

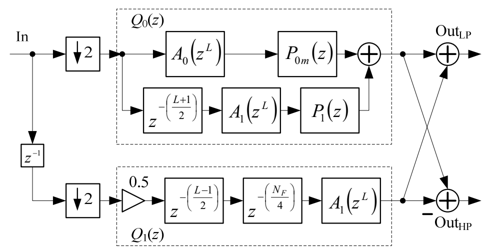

Fig. 4. Efficient realization structure of the analysis bank.

Fig. 5 plots the frequency responses of the low-pass half-band model filter $G(z)$ designed for $\omega_p^{(G)}$ =0.4 ($\omega_s^{(G)}$ =0.6). The upper subfigure of Fig. 6 shows the gain responses of periodic model filter pair [$G(z^5)$ $G_c(z^5)$]. The plots of masking filters gain responses, given in the lower subfigure of Fig. 6, illustrate the accordance with the sketch given in Fig. 2(c). The frequency response details are given in Fig. 7 for the low-pass branch, while Fig. 8 plots the gain responses of the two-channel filters and the magnitude reconstruction property of the bank.

The analysis bank of this example can be implemented by making use of the efficient implementation structure as given in Fig. 4. In that case, only 24 multiplication constants are needed: (i) 5 multipliers for $A_0(z)$, whereas $A_1(z)$ is a delay ($A_1(z) = z^{-4}$); (ii) 10 multipliers for P_{0m}(z) and 9 multipliers for P_1(z) when exploiting the coefficient symmetry in P_{0m}(z) and P_1(z). The overall delay of the FRM based filter pair of this example amounts to 63 samples, see Fig. 7.

An alternative solution can be achieved with QMF FIR model filters [4], [10]. In that case, masking filters of this example can be used, whereas the 47th order FIR model filter is required. For L = 5, the delay of the FIR periodic model filter amounts to 117.5 samples, and the total delay of the filter pair is 135.5 samples; more than two times the delay achieved with the proposed IIR filter pair.

VI. Conclusion

In this paper, we have considered the performances and computational efficiency of two-channel filter banks based on the frequency-response masking technique where the model filter is an IIR half-band filter. An efficient realization structure for the FRM-based analysis bank is developed. It is shown by means of an example that high channel selectivity can be achieved with an approximately linear phase in the sub-channels. When compared with the alternative solutions based on FIR model filters, the proposed solution reduces the arithmetic complexity, and the overall delay of the bank is more than halved.

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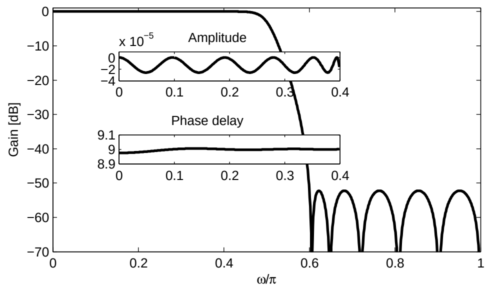

Fig. 5. Frequency response of the low-pass model filter $G(z)$.

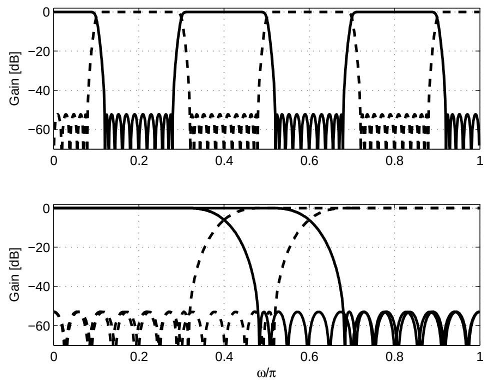

Fig. 6. Frequency responses of: periodic model filter pair; masking filters

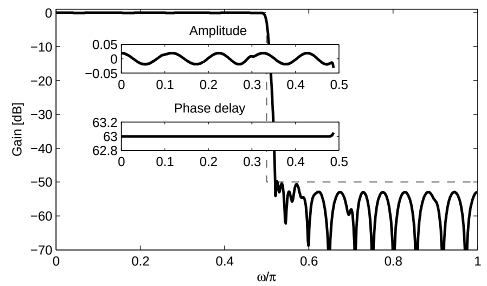

Fig. 7. Frequency responses of low-pass filter $H_{a0}(z)$.

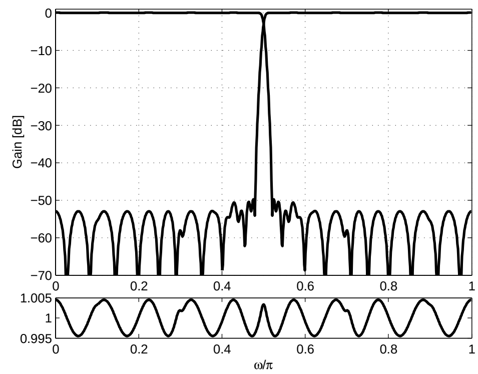

Fig. 8. Frequency responses of low-pass/high-pass filter pair [$H_{a0}(z)$, $H_{a1}(z)$], and verification of nearly perfect power-reconstruction property.