Performance Evaluation of STBC MIMO Systems with Linear Precoding

Ancuţa Moldovan, Tudor Palade, Emanuel Puşchiţă, Irina Vermeşan, and Rebeca Colda

***Abstract* — It is known that transmit channel side information (CSIT) is used to enhance the performance of space-time block codes based multi-antenna communication links. In this paper, we analyze how transmission algorithms can be adapted to the channel condition based on the degree of the available CSIT and the system diversity order. The precoding design criterion considered is minimizing the average pairwise error probability. The analyzed parameters are the bit error rate (BER) and the link throughput.**

***Keywords* — CSIT, precoding, STBC, waterfilling.**

I. Introduction

Conventional SISO (Single Input Single Output) systems are limited by the multipath propagation and interference, so they cannot satisfy the demand for high data rates and better quality systems [1]. The benefits of MIMO (Multiple Input Multiple Output) techniques are well established: linear growth in transmission rate with the minimum number of antennas, enhanced link reliability and coverage, efficient use of bandwidth, are all obtained without additional radio resources requirements, like bandwidth or more transmit power. The only demand is that the receiver has perfect knowledge of the channel state information (CSIR – Channel State Information at the Receiver).

Exploiting channel state information at the transmitter can provide further enhancement in the performance of a MIMO system, regarding both the channel capacity and the system error performances, even if the channel is spatially correlated. These adaptive techniques allow the transmitter to adapt to the propagation conditions on the channel [2].

The rest of the paper is organized as follows: in Section II the system model with STBC (Space-Time Block Codes) and CSIT is presented. In Section III we introduce the design of the linear precoder for three different types of CSIT (full, mean, covariance), under the constraint of a fixed value transmit-power. Section IV contains the results of the simulations and in Section V the conclusions are drawn.

II. System Model

A frequency-flat Rayleigh fading MIMO wireless channel with N_T transmit and N_R receive antennas is considered. The system is encoded with a STBC: the incoming bits b_i are mapped onto a vector $s=[s_0,s_1,...,s_{N_s-1}]^T$, where s_i is a symbol from a uniform signal constellation such as M-QAM, M-PSK or M-PAM [3]. The generated symbols are encoded in two dimensions, space and time, according to a OSTBC design matrix $C(s)[N_s \text{x} N]$, where N_s and N are the time and space dimensions. The codeword matrix [4, p85] is then processed by a precoder $F[N_T \text{x} N_s]$, designed according to the available CSIT. The $N_R \text{x} N$ received signal becomes:

$$Y = HFC(s) + n \tag{1}$$

where $n[N_R \text{x} N]$ is the additive white noise and $H[N_R \text{x} N_T]$ is the channel matrix.

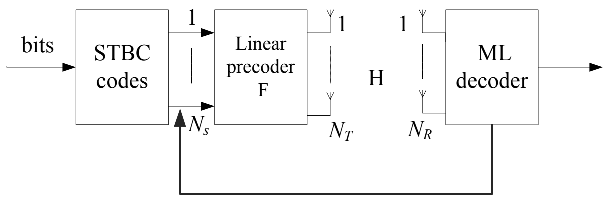

Fig. 1. STBC MIMO system with CSIT.

At the receiver, which is assumed to have perfect knowledge of channel state, a MLD (Maximum Likelihood Decoding) detection is performed in order to estimate the transmitted symbols [5]:

$$\overline{C} = \arg\min_{C} \| Y - HFC \|_F^2 \tag{2}$$

The design of the precoding matrix depends on the degree of available CSIT and the performance criteria that are considered: maximizing the system ergodic capacity, minimizing the pair-wise error probability (PEP), the symbol error probability (SER) or the mean squared error (MSE) [5]. For the simulations in this article, minimizing the pair-wise error probability is considered.

III. Precoding Design

The general form of a linear precoder is given below:

$$F = U_F D V_F^H \tag{3}$$

which is the singular value decomposition of the matrix. The left singular vectors U_F give the orthogonal beam directions, the beam power loadings are the squared singular values D^2 and, V_F, the right singular vectors, is the input shaping matrix as in [5]. The constraint that the matrix has to satisfy is that the sum of power over all beams must be constant, $tr(FF^*)=1$.

The authors are with the Department of Communications, Technical University of Cluj-Napoca, Romania, 26-28 Bariţiu Street (e-mail: Ancuta.Moldovan@com.utcluj.ro)

A. Full CSIT

If the entire channel matrix is available at the transmitter, the precoder is based on the channel matrix H and on the input codeword covariance matrix Q as in [5]:

$$F = V_H \Lambda_f U_Q \tag{4}$$

where $V_H[N_R \text{x} N_R]$ is obtained by singular value decomposition of the channel matrix and $U_Q[N_s \text{x} N_s]$ is obtained by eigenvalue decomposition of the covariance matrix Q. The optimal power allocation, given in [5], is through water-filling and the power division depends on the eigenvalues of the input codeword covariance and the eigenvalues of the channel:

$$p_i = (\lambda - \frac{N_0}{\lambda_i(HH^H)\lambda_i(Q)})_+ \tag{5}$$

where λ is the Lagrange multiplier chosen to satisfy the power constraint and N_0 is the noise power per spatial dimension.

B. Mean CSIT

For this type of feedback the MIMO channel matrix is given by:

$$H = \overline{H} + \Xi \tag{6}$$

where $\overline{H}[N_R \text{x} N_T]$ is the CSIT estimated at the transmission and $\Xi[N_R \text{x} N_T]$ is the CSIT error matrix. These two matrices are uncorrelated $E[\overline{H}\Xi^H] = 0$, their entries are zero-mean complex Gaussian with the variances satisfying $\sigma_e^2 + \sigma_h^2 = 1$. The optimal precoder is given in [6]:

$$F = U_h \Lambda_f V_e^H \tag{7}$$

where $U_h[N_T \text{x} N_T]$ is obtained by singular value decomposition of the matrix $\overline{H}$ and $V_e[N_s \text{x} N_s]$ is obtained by eigenvalue decomposition of the codeword error matrix $E(m,n)=[s_m - s_n]$, which is the error probability of choosing the nearest space-time codeword s_n instead of the transmitted codeword s_m [6]. The power allocated to each subchannel is computed based on the eigenvalues of the estimated channel matrix $\Lambda_h[N_R \text{x} N_T]$ and of the codeword error matrix $\Lambda_e[N_s \text{x} N_s]$, and also on the noise variance σ_n^2. The value is given by:

$$p_k^* = (\frac{\sqrt{\frac{\lambda_{h,k}^2 \lambda_{e,k}}{\lambda}} - 1}{\sigma_n^2 \lambda_{e,k}})_+ \tag{8}$$

where λ is the Lagrange multiplier.

C. Covariance CSIT

When the channel covariance matrix is available at the transmitter, the optimal precoder design that minimizes the maximum pairwise error probability is given in [7] as:

$$F = \frac{1}{\sqrt{\eta}} U_T \sqrt{B} \tag{9}$$

The precoder matrix depends on the correlation between the transmit antennas $R_T = U_T \Lambda_T U_T^H$, on the noise variance σ_n^2 and a scaling factor μ_{kl} that depends on the codeword matrix at the transmission C_k and the codeword matrix after detection C_l:

$$\eta = \frac{\mu_{\min}}{4\sigma_n^2} \tag{10}$$

$$\mu_{\min = \arg\min_{\mu_{kl}}} \{\mu_{kl} I = (C_k - C_l)(C_k - C_l)^H\} \tag{11}$$

The power allocation is an extension of the waterfilling problem to two dimensions (if the correlation between the receive antennas is also considered), and the matrix solution is given in [7]:

$$B_{opt} = \underset{\substack{B \geq 0 \\ tr(B) = \eta N_T}}{\arg\max} \det[I_{N_R} \otimes B + \Lambda_R^{-1} \otimes \Lambda_T^{-1}] \tag{12}$$

which is equivalent with finding non-negative b_m values that maximize $\prod_{m=1}^{N_T} \prod_{n=1}^{N_R} \left(b_m - \lambda_{rn}^{-1} \lambda_{tm}^{-1}\right)$, Λ_T and Λ_R are the eigenvalues of the transmit, respectively receive, antennas correlation matrices. For a MISO system the complexity of the computation is significantly reduced, $b_i = \max\left(\nu - \lambda_{ti}^{-1}, 0\right)$, $i=1,\ldots,N_T$ and ν is a constant chosen to satisfy the power constraint.

IV. Link Adaptation

Space-time block codes, that were considered in this article as a transmission technique, provide full spatial diversity, but they cannot improve the peak error free data throughput only if they are combined with AMC (Adaptive Modulation and Coding).

The IEEE 802.11a standard defines a range of modulation and coding schemes at the PHY layer. Based on these, different MIMO configurations can be applied in order to achieve a certain link speed [8]. The link throughput is estimated from the PER (Packet Error Rate) as follows:

$$R = D(1 - PER) \tag{13}$$

where D is the transmission data rate defined as $D=(N_D N_b R_{FEC} R_{STC})/T_S$, N_D is the number of data subcarriers, N_b the coded bits per subcarrier, R_{FEC} is the forward error correction (FEC) coding rate, R_{STC} is the space-time coding rate and T_S is the OFDM symbol duration. Based on the channel information at the transmission side, the transmitter is able to select between different PHY modes in order to fully exploit the available radio resources and to maximize the link speed.

V. Simulation Results

We provide simulations for MIMO systems with a varying number of transmit/receive antennas, based on different types of CSIT. The transmit symbols are uniformly distributed based on a M-PSK/M-QAM constellations. The transmit power, across all transmit antennas, is set to one. Regarding the channel, a frequency flat, quasi-static MIMO channel is considered.

The purpose of these simulations is to see when it is efficient to use CSIT and which amount of feedback information is needed to obtain a certain performance, for different configurations of MIMO systems.

The simulations are realized using Matlab 7. The analyzed parameters are the bit error rate and the throughput.

A. BER performance

For the first simulations that were done we considered no receive diversity, a system with two transmit antennas and one receive antenna, encoded with the rate one Alamouti space-time code. By considering this configuration it can be analyzed how the performance of the MIMO system can be improved by acting only at the transmission side. For the first two types of CSIT feedback the transmit antennas are supposed to be uncorrelated.

In Fig. 2 there are the results obtained if the transmitter is assumed to have full channel state knowledge. With a small diversity order, equal to 2, there is a precoding gain of 5 dB for the entire SNR (Signal to Noise Ratio) domain that is analyzed.

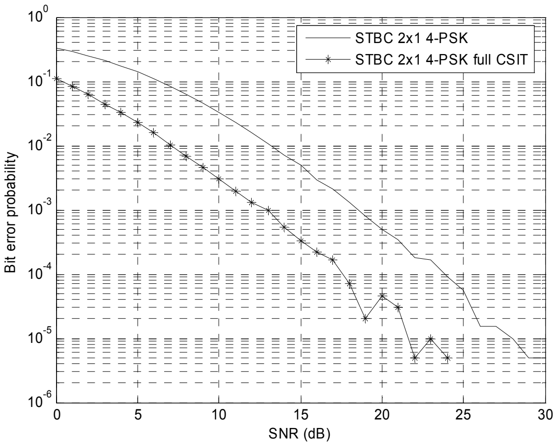

Fig. 2. Precoding gain for a 2x1 STBC MIMO system with full CSIT.

If the transmitter has only statistical information regarding the channel, mean based estimation with a small CSIT error σ_e^2=0.01, the precoding gain is smaller, about 3 dB, but it still outperforms the transmission with no CSIT as it can be observed in Fig. 3. For high CSIT estimation errors at the transmitter, σ_e^2=0.1, the error performances of the transmission are enhanced only for small values of the SNR (Signal to Noise Ratio).

In the next simulation we analyze the performance of a covariance based CSIT transmission with one receive antenna employing a rate one space-time block code. First of all it can be noticed that the spatial correlation dramatically affects the performance of a STBC transmission with no CSIT. For low values of SNR, smaller than 6 dB, the performances of non-coded transmission are similar, but above this threshold, diversity is essential and the non-correlated system outperforms the transmission when there is a correlation of ρ=0.9775 between the two transmit antennas. It can be noticed that the BER curve is different, as the diversity order is different.

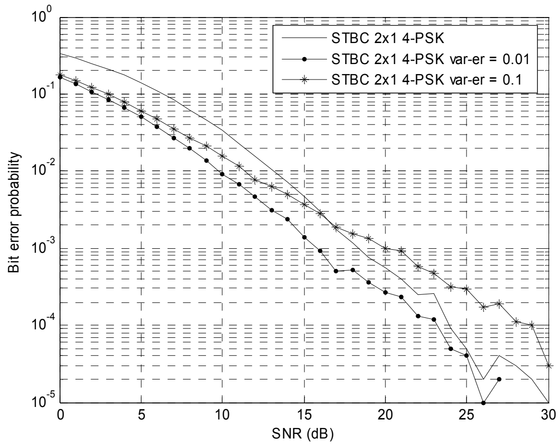

Fig. 3. Precoding gain for a 2x1 STBC MIMO system with mean CSIT.

If the transmit antennas are correlated (ρ=0.9775), but the system benefits from CSIT, there is a precoding gain of about 7 dB for the entire SNR region, as it is depicted in Fig. 4. So if the channel is correlated, adapting the transmission based on available covariance CSIT leads to a significant error performance improvement.

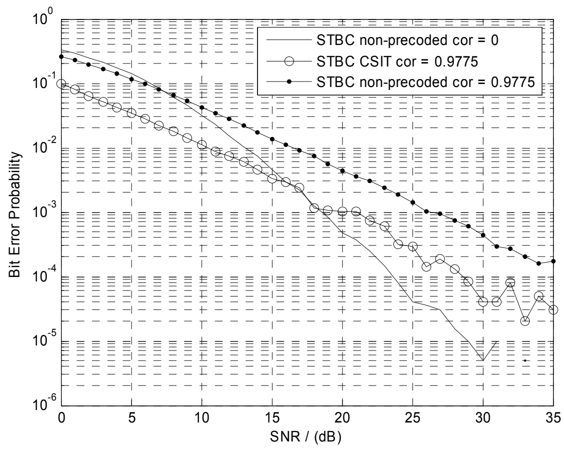

Fig. 4. Precoding gain for a 2x1 STBC MIMO system with covariance CSIT.

In Table 1 there are the values of the SNR needed for various configurations of MIMO system with and without CSIT to ensure a certain performance level regarding the bit error rate. The influence of the amount of information regarding the channel state available at the transmitter and the influence of transmit and receive diversity can be analyzed from the results summarized in this table. By increasing the number of receive antennas, N_R=2, the SNR gain is by about 10 dB for both transmissions, with and without CSIT. We will compare the 2x2 MIMO system with the systems obtained by adding additional receive and transmit antennas: 2x4 and 4x2.

For the system with no CSIT, receive diversity is more important to achieve a better error performance. A rate one transmission is 3 dB better than the same diversity order transmission, obtained with four transmit antennas. This observation is not true for the systems with CSIT, especially if the precoder design is based on statistical CSIT. The 2x4 mean based configuration requires a

TABLE 1: PERFORMANCE RESULTS FOR DIFFERENT DIVERSITY ORDER MIMO SYSTEMS.

System	***2x1***		***2x2***		***2x4***		***4x2***		***4x4***	
BER	10^{-4}	10^{-5}	10^{-4}	10^{-5}	10^{-4}	10^{-5}	10^{-4}	10^{-5}	10^{-4}	10^{-5}
No CSIT	24 dB	27 dB	14 dB	17 dB	8 dB	10 dB	11 dB	13 dB	7 dB	8 dB
Full CSIT	17 dB	22 dB	7 dB	9 dB	4.5 dB	6.5 dB	4 dB	6 dB	0.5 dB	1.5 dB
Mean CSIT σ_e^2=0.01	22 dB	26 dB	13 dB	15 dB	7 dB	9 dB	5 dB	7 dB	3.2 dB	4.5 dB
Mean CSIT σ_e^2=0.1	26 dB	>30 dB	13.5 dB	16 dB	7.5 dB	9.5 dB	6 dB	8 dB	3.7 dB	5 dB
Covariance CSIT	29 dB	35 dB	14 dB	18 dB	-	-	-	-	-	-

SNR=9 dB for *BER*=10^{-5}, while the 4x2 system only requires a *SNR*=7 dB to obtain the same performance, but the limitation of the orthogonal space-time block code with 4 transmit antennas is its rate of 1/2. The error performance improvement is due to the higher precoding gain that can be obtained by optimal antenna selection, beamforming and power loading.

B. Throughput performance

Choosing the MIMO configuration that efficiently uses the available radio resources has to be done according to the user/application requirements. We intend to determine the most appropriate configuration at the physical layer, based on predefined performance criteria. Regarding the packet length, so far, only fix values have been considered.

In the first part of the simulation, as physical layer parameters we only considered QPSK signal constellation and constant diversity order MIMO systems. Perfect channel knowledge at the receiver is assumed, while for the transmitter two cases are evaluated: no information regarding the channel state is available at the transmitter and partial CSI based on channel mean, with small CSIT estimation errors, is used for precoding. In Fig. 5 and Fig. 6 there are the packet error rate and the throughput performances for the considered systems.

If we compare the 2 transmit antennas systems, it can be noticed that there is only 1 dB precoding gain that improves the performances when CSIT is available at the transmitter. For a high diversity order at the transmitter, CSIT leads to an improvement of 6 dB, as the spatial resources are efficiently used.

Regarding the throughput, a FEC rate of ¾ was considered, N_D=48 data subcarriers and T_S=4 µs. The two transmit antennas systems lead to the highest link speeds due to the rate one codeword orthogonal design.

From the user's perspective, for a fix modulation rate, diversity at the transmitter contributes to the best error performances through adapting the beams directions and the power allocated to each beam. Due to the rate penalty suffered by the codeword matrices with more than two transmit antennas, for applications that require a high link speed, two transmit antennas systems are to be used.

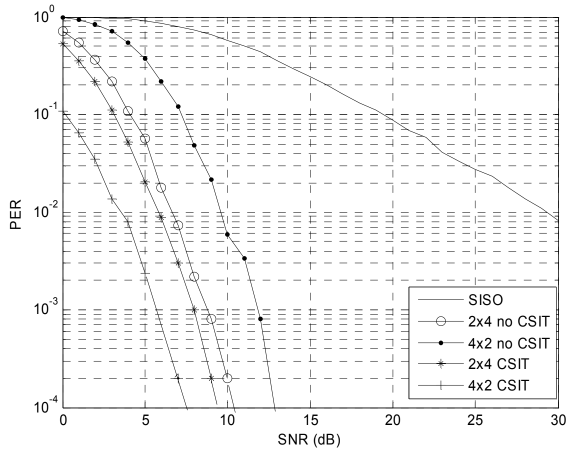

Fig. 5. PER performances for transmissions with the same constellation size.

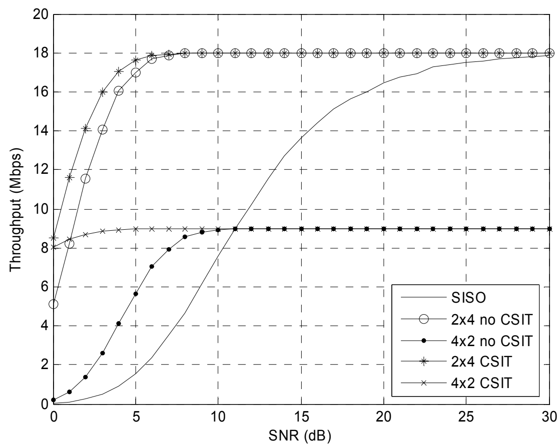

Fig. 6. Throughput performances for transmissions with the same constellation size.

In the next simulation we considered the same performance criteria and the transmissions are characterized by the same diversity order and the same spectral efficiency, η=2 bits/s/Hz. To do this, an appropriate modulation scheme has to be chosen for each space-time codeword matrix. The rate one space-time code is combined with 4-PSK modulation, while, the rate penalty suffered by the 4 transmit antennas system, has to be compensated with a higher order modulation - 16-QAM.

If we compare the error performances in Fig. 5 and Fig. 7, the 4 transmit antennas system suffers a depreciation of 9 dB at a $PER=10^{-3}$ due to a higher error sensitivity of 16-QAM modulation. If the system benefits from a precoding gain, the SNR difference is only 6 dB.

For low values of the SNR, the 4 transmit antennas system with channel estimation at the transmitter leads to the best error performances, due to an efficient use of the radio resources. For medium and high values of the SNR, the power is allocated only to the strongest eigenvalues of the channel, so the transmission has a lower diversity order.

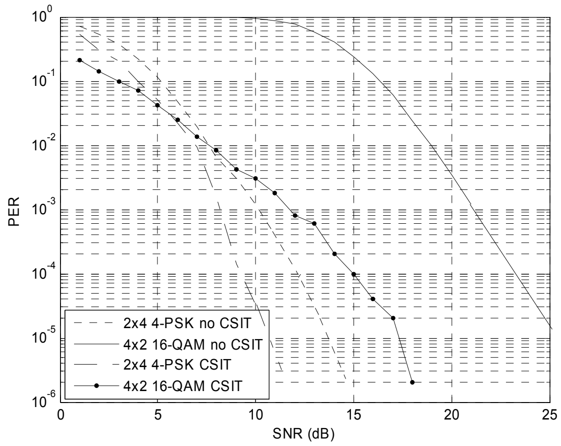

Fig. 7. PER performances for transmissions with the same spectral efficiency.

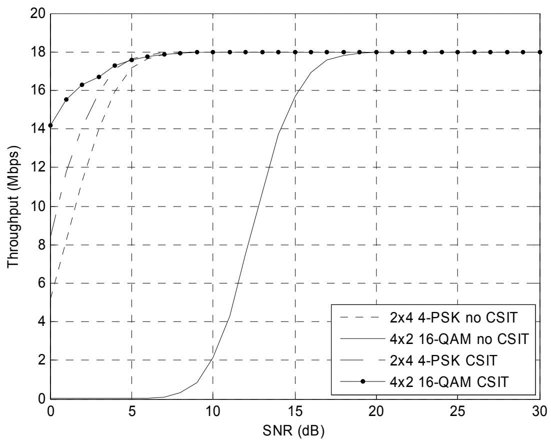

Fig. 8. Throughput performances for transmissions with the same spectral efficiency.

Regarding the throughput for low values of the SNR the 4x2 system with CSIT significantly improves the link speed compared to the 2x4 system with CSIT. The difference between the two transmissions is about 6 Mbps.

VI. Conclusions

The paper evaluates the potential benefits that can be obtained by means of precoding in a MIMO system. By exploiting channel knowledge at the transmitter, the channel capacity and the system error performance are significantly improved even if the system is spatially correlated. Choosing the right form of CSIT depends on the MIMO channel characteristics and the feedback channel rate and delay. If the channel is slow time varying, the precoding based on the mean channel matrix estimation can be implemented, but if the channel is fast time varying, covariance based CSIT is more appropriate.

Statistical channel information (mean or covariance) available at the transmitter is used to adapt the transmission to the channel state. The transmit parameters that can be controlled are the beam direction, the power allocated to each beam, the modulation scheme and the coding rate.

If CSIT is available, a higher number of transmit antennas ensures better error and throughput performances, as the radio resources are efficiently distributed.

For fix spectral efficiencies, the higher gain compared to the transmission with no CSIT is obtained for higher diversity orders even if the code rate is smaller.

It must be stated that all the transmissions with CSIT outperform the STBC transmissions with no channel adaptability.

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