Accurate BEP of OSTBC-MIMO Systems with DF Incremental Relay Selection

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***Abstract* — This paper addresses the analysis of Bit Error Probability (BEP) in Multiple Input Multiple Output (MIMO) systems based on Orthogonal Space Time Block Coding (OSTBC) and Incremental Decode and Forward (IDF) protocol with relay selection. According to IDF, the direct link is used by the destination, if it can successfully decode the source codeword; otherwise, it requests the assistance of the best relay among those that have correctly decoded the received codeword. In the relaying phase, the best relay decodes the source codeword and forwards the received symbols to the destination. The proposed numerical results show that the BEP of OSTBC-MIMO systems with relay-selection IDF is enhanced compared to the case without relay selection. These results are obtained assuming that the channel fading envelopes of different links are independent Rayleigh distributions.**

***Keywords* — cooperative system, decode and forward, incremental relaying, MIMO, OSTBC, relay selection.**

I. Introduction

IN the last decade, the analysis of cooperative wireless networks has attracted great research interest thanks to its potential to increase the channel capacity and to improve wireless links reliability. The fundamental idea of cooperative networks is the ability to use relays to provide space distributed antennas. The relays assist a source to transmit information to its destination by sending multiple copies of its symbols through independent paths [1]. The main considered relay models may be either regenerative such as decode and forward (DF) or non regenerative such as amplify and forward (AF). According to DF and AF protocols, the relay forwards the correctly decoded signal and the amplified received signal, respectively.

In the literature, several studies have focused on the performance analysis of cooperative systems, such as [2]-[4] where the Symbol Error Probabilities (SEP) of cooperative AF and DF relaying are analyzed.

In regular one-way DF systems, regardless of channel conditions, the transmission of each symbol requires two phases: the source broadcast phase and the relaying phase. Hence, the available degrees of freedom of the space could not be used efficiently. In order to overcome this drawback and to improve the performances of conventional relay protocol, incremental relaying was proposed, in [5] [6] for example.

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Incremental cooperative systems are considered to be an interesting way to save the resources of wireless networks by improving the spectral efficiency and restricting the use of the relaying phase only when necessary. Several studies have dealt with incremental relaying protocols, such as [5]-[7], where the exact BEP of incremental amplify-and-forward and decode-and-forward protocols have been derived.

To enhance the throughput of IDF systems, the use of best relay selection has been shown to be an attractive solution, as presented in [8]-[12]. In [10]-[11], the authors investigate the exact expressions of the BEP and the outage probability of cooperative IDF systems.

Many studies of incremental cooperative-selective communication systems have focused on networks with multiple relays where all nodes are equipped by a single transmit and receive antenna.

Besides the interesting studies of incremental cooperative systems, the OSTBC – MIMO systems have been intensively studied [13]-[14]. These schemes provide an interesting spectral efficiency and reliability. If incremental cooperative techniques are used in OSTBC-MIMO systems, the network coverage and the end to end channel reliability and throughput could be enhanced [15].

If incremental-relay selection is used in OSTBC-MIMO, the end to end system performance could be effectively enhanced mainly in throughput and coverage. Indeed, in this paper, we extend the work of our paper presented at Telfor'2012 [16], where the performances of an incremental MIMO DF relaying with one relay are analyzed. The work in [16] is extended to the case of multiple relay networks with the best relay selection. The relay that should be selected is the one owning the maximum SNR among the set of relays that correctly decoded the received signals. Hence, our contribution in this paper consists of the performance analysis of the scheme of incremental DF relay selection OSTBC-MIMO networks where all transmissions are done over independent distributed Rayleigh fading channels. Thus, we derive the accurate end to end BEP at the destination and we study the effect of the signal to noise ratio (SNR) threshold on system performances. We also compare performances offered by the incremental-selective MIMO system with the same system without relay selection and with the incremental-selective SISO one.

The remainder of this paper is organized as follows. Section II presents the system and channel models. Section III describes a theoretical analysis of the BEP at the destination of DF incremental-relay selection in OSTBC-

MIMO networks. A selection of numerical results is presented and discussed in Section IV.

II. System and Channel Model

As shown in Fig. 1, a one-way-two-hop multi-relay MIMO system is considered. This system is composed of a pair of source-destination and M DF relays denoted r_i (i=1,...,M). Thus, the source has n_{tr}^{s} transmit antennas, each relay has $n_{rc}^{r_i}$ receive antennas and a single transmit antenna and the destination has n_{rc}^{d} receive antennas.

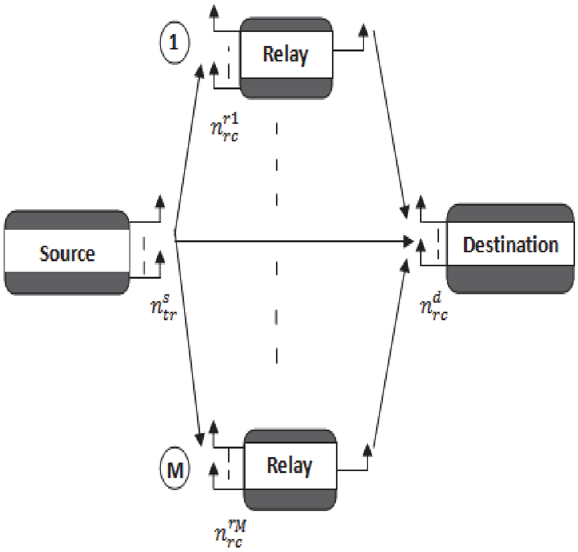

Fig. 1. System model.

All transmissions are made over orthogonal channels using three phases:

Phase 1: The source broadcasts an OSTBC data codeword to the M relays and the destination through its n_{tr}^{s} antennas. All relays and the destination receive faded and noisy versions of the source signal. The received signals at the destination and the *i-th* relay node, denoted Y_{sd} and Y_{sr_i}, which are given by:

$$Y_{sd} = \sqrt{\frac{E_s}{n_{tr}^{s}}}\, H_{sd}X + N_{sd}$$

$$Y_{sr_i} = \sqrt{\frac{E_s}{n_{tr}^{s}}} H_{sr_i}X + N_{sr_i}$$

where E_s is the transmitted energy per symbol by the source, N_{yz} is an additive white complex Gaussian noise (AWGN) with a variance N_0 and H_{yz} is the channel matrix of (y-z) link assuming that each element is an i.i.d. complex Gaussian random variable with zero mean and variance σ_{yz}. H_{yz} can be written as:

$$H_{yz} = \begin{pmatrix} h_{yz}^{(1\,1)} & \cdots & h_{yz}^{(1\, n_{tr}^{y})} \\ \vdots & \ddots & \vdots \\ h_{yz}^{(n_{rc}^{z}\,1)} & \cdots & h_{yz}^{(n_{rc}^{z}\, n_{tr}^{y})} \end{pmatrix}$$

X is the code matrix which depends on the number of transmit antennas of the source, which can be illustrated in the following example, if n_{tr}^{s}=2, the Alamouti code is used:

$$X = \begin{pmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{pmatrix}$$

where (.)* denotes a complex conjugate of (.).

Phase 2: By comparing the received direct link SNR to a given threshold γ_{th}, the destination decides whether it should be assisted by a relay or not. We note that γ_{th} represents the minimum SNR for which the destination can detect the signal successfully without the need of the relayed signal [17]. If the instantaneous SNR of the direct link (s-d) is greater than γ_{th}, then all the relays remain quiet and the source transmits a new codeword, otherwise the best relay forwards a correctly decoded symbol with symbol energy $E_{r_{sel}}$ to the destination. Thus, the destination selects the relay belonging to Z having the largest SNR in $(r_Z - d)$ links where Z is the set of relays that have correctly decoded the received signal and r_Z denotes the relays in the set Z.

The received signal at the destination from the selected relay, $Y_{r_{sel}d}$, can be written as:

$$Y_{r_{sel}d} = \sqrt{E_{r_{sel}}} H_{r_{sel}d}\hat{X} + N_{r_{sel}d}$$

where $\hat{X}$ = x_i the correctly decoded symbol by the selected relay, and $H_{r_{sel}d}$ is an N_{rc}^{d}*1 channel vector.

Phase 3: If the system has used the relaying link, the destination combines the received signals coming from both the source and the selected relay using the Maximum Ratio Combining (MRC), otherwise it decodes the received OSTBC codeword transmitted by the source, using a Maximum Likelihood (ML) detector.

III. Theoretical Analysis

The average BEP at the destination of the incremental and selective MIMO relaying protocol, P_{Inc}, can be expressed as:

$$P_{Inc} = Pr(\gamma_{sd} \le \gamma_{th})\ P_{DF} + Pr(\gamma_{sd} > \gamma_{th})\ P_{dir}$$

where P_{DF} and P_{dir} represent, respectively, the end to end average BEP at the destination when the selected relay cooperates with the source by transmitting a second copy of the source signal and the average BEP at the destination for the direct link.γ_{th}, previously defined, is the SNR threshold and γ_{sd} is the instantaneous SNR between the source and the destination which can be expressed as:

$$\gamma_{sd} = \sum_{m=1}^{n_{rc}^{d}} \sum_{k=1}^{n_{tr}^{s}} \gamma_{sd}^{(mk)} \quad (1)$$

where

$$\gamma_{sd}^{(mk)} = \frac{E_s}{n_{tr}^{s}\, N_0} \left|h_{sd}^{(mk)}\right|^2$$

$\gamma_{sd}^{(mk)}$ is the SNR between the *m-th* destination's receive antenna and the *k-th* source's transmit antenna. $h_{xy}^{(mk)}$ represents the channel coefficient between the *m-th* Rx antenna of y and the *k-th* Tx antenna of x. Assuming that γ_{xy}, the instantaneous SNR of the (x-y) link and follows the Rayleigh distribution, the probability that γ_{xy} is lower than γ_{th} can be written as:

$$Pr(\gamma_{xy} \le \gamma_{th}) = \int_0^{\gamma_{th}} f_{\gamma_{xy}}(\gamma) d\gamma \quad (2)$$

where $f_{\gamma_{xy}}(\gamma)$ is the Probability Density Function (PDF) of γ_{xy} which is defined in [15] as:

$$f_{\gamma_{xy}}(\gamma)=\varphi(\gamma,a,l)=\frac{\gamma^{l-1}}{a^{l}(l-1)!}\,e^{\frac{-\gamma}{a}},\gamma\geq 0 \quad (3)$$

γ_{xy} is the SNR of the (*x*-*y*) link which can be calculated as in (1).$\varphi(\gamma,a,l)$ is the chi-square distribution with $2l$ degrees of freedom and average mean $a\times l$ where $a=\bar{\gamma}_{xy}^{(mk)}$, $l=n_{rc}^{y}\times n_{tr}^{x}$ and $\bar{\gamma}_{xy}^{(mk)}$ is:

$$\begin{aligned}\bar{\gamma}_{xy}^{(mk)}&=E(\gamma_{xy}^{(mk)})\\&=E(\frac{E_x}{N_0 n_{tr}^{x}}\left|h_{xy}^{(mk)}\right|^2)\\&=\frac{E_x}{N_0 n_{tr}^{x}}E(\left|h_{xy}^{(mk)}\right|^2)\\&=\frac{E_x}{N_0 n_{tr}^{x}}\frac{1}{d_{xy}^{\alpha}}\end{aligned}$$

where $E(.)$ is the expectation operator, α is the path loss exponent and d_{xy} is the normalized distance between x and y. Note that $d_{xy}=\frac{d_{xy}^{eff}}{d_0}$, d_{xy}^{eff} is the effective distance in meters between x and y, d_0 is an arbitrary reference distance.

Substituting (3) into (2) and after doing some simplifications, we have this expression of $Pr(\gamma_{xy}\leq\gamma_{th})$:

$$\begin{aligned}Pr(\gamma_{xy}\leq\gamma_{th})&=\frac{1}{(l-1)!}(\Gamma(l)-\Gamma(l,\frac{\gamma_{th}}{a}))\\&=1-\frac{\Gamma(l,\frac{\gamma_{th}}{a})}{\Gamma(l)}\end{aligned} \quad (4)$$

where $\Gamma(l)$ is the gamma function which is given by: $\Gamma(l)=\int_0^{\infty}e^{-t}\,t^{l-1}\mathrm{d}t$ and $\Gamma(.,.)$ is the lower incomplete gamma function.

On the one hand, for the BPSK modulation, the average error probability of the direct link can be calculated as:

$$P_{dir}=\int_0^{\infty}\frac{1}{2}\,erfc(\sqrt{\gamma})\;f_{\gamma_{sd}}(\gamma|\gamma_{sd}>\gamma_{th})d\gamma$$

where $erfc()$ is the complementary error function which is expressed as:

$$erfc(y)=\frac{2}{\sqrt{\pi}}\int_y^{\infty}e^{-t^2}dt$$

$f_{\gamma_{sd}}(\gamma|\gamma_{sd}>\gamma_{th})$, the PDF of γ_{sd} checking the condition where γ_{sd} is larger than the threshold γ_{th}, can be obtained as follows:

$$f_{\gamma_{sd}}(\gamma|\gamma_{sd}>\gamma_{th})=\begin{cases}0 & ,\gamma\leq\gamma_{th}\\ -\frac{(\frac{\gamma}{a})^{l-1}e^{\frac{-\gamma}{a}}}{a(l-1)!} & ,\gamma>\gamma_{th}\end{cases}$$

Proof: Generally, to calculate $f_{\gamma_{xy}}(\gamma|\gamma_{xy}>\gamma_{th})$, we follow these stages:

$$\begin{aligned}Pr(\gamma_{xy}>u)&=\int_u^{\infty}\frac{\gamma^{l-1}}{a^{l}(l-1)!}\,e^{\frac{-\gamma}{a}}\,d\gamma\\&=\frac{1}{(l-1)!}\Gamma(l,\frac{u}{a})\\&=\frac{\Gamma(l,\frac{u}{a})}{\Gamma(l)}\end{aligned} \quad (5)$$

$f_{\gamma_{xy}}(\gamma|\gamma_{xy}>\gamma_{th})$ is obtained by deriving the result in (5) with respect to u. Thus, P_{dir} can be written as:

$$P_{dir}=\int_{\gamma_{th}}^{+\infty}\frac{1}{2}erfc(\sqrt{\gamma})\left(-\frac{(\frac{\gamma}{a})^{l-1}e^{-\frac{\gamma}{a}}}{a(l-1)!}\right)d\gamma.$$

After a deep calculation of this integral, we finally obtain this expression of P_{dir}:

$$P_{dir}=-\frac{1}{2(l-1)!}(erfc(\frac{1}{\sqrt{\gamma_{th}}})(\sqrt{a\gamma_{th}}\times\Gamma(\frac{1}{2}+l,\frac{\gamma_{th}}{a}))).$$

On the other hand, P_{DF} can be calculated as:

$$P_{DF}=\sum_{Z}Pr(Z)P_{DF|Z}$$

$Pr(Z)$ is the probability that Z is the set of relays that have correctly decoded the received signal. It can be written as:

$$Pr(Z)=\prod_{i\in Z}Pr(\gamma_{sr_i}>\gamma_{th})\times\prod_{j\notin Z}Pr(\gamma_{sr_j}\leq\gamma_{th})$$

$Pr(\gamma_{sr_j}\leq\gamma_{th})$ and $Pr(\gamma_{sr_i}>\gamma_{th})$ were respectively defined in (4) and (5).

If Z contains more than one relay, the relay having the largest SNR will be activated. So, the SNR of $(r_{sel}-d)$ link is given by:

$$\gamma_{r_{sel}d}=\max_{i\in Z}\{\gamma_{r_id}\}$$

$P_{DF|Z}$ can be written as:

$$P_{DF|Z}=\sum_{q\in Z}P_{DF|r_{sel}}Pr(r_{sel=q})$$

where r_{sel} is the selected relay in Z. The probability $Pr(r_{sel=q})$ is given by:

$$Pr(r_{sel=q})=\prod_{\substack{k\in Z\\k\neq q}}Pr(\gamma_{r_qd}>\gamma_{r_kd}).$$

According to equation (5), $Pr(r_{sel=q})$ can be finally expressed as:

$$Pr(r_{sel=q})=\prod_{\substack{k\in Z\\k\neq q}}\frac{\Gamma(n,\frac{\gamma_{r_kd}}{b})}{\Gamma(n)}$$

with $b=\bar{\gamma}_{r_qd}^{(mk)}$, $n=n_{rc}^{d}*n_{tr}^{r_q}$.

When the system used the relaying link, the BEP at the destination can be presented in the following equation:

$$P_{DF|r_{sel}}=\prod_{k\notin Z}P_{sr_k}\times P_{sd}+\prod_{k\in Z}P_{sr_k}\times P_{d|r_{sel}}$$

where

- $P_{d|r_{sel}}$ is the BEP at the destination when the selected relay sends another copy of the received signal. In appendix, we show that $P_{d|r_{sel}}$ can be derived as in [15]:

$$P_{d|r_{sel}}=\sum_{i=1}^{n_{tr}^{s}n_{rc}^{d}}\delta_i\theta(i,s,d)+\sum_{i=1}^{n_{rc}^{d}}\lambda_j\theta(j,r_{sel},d)$$

where δ_i and λ_j are the residues which were computed similarly to [15]. $\theta(p,v,w)$ is given by:

$$\theta(p,v,w)=A\left[\frac{1-\mu_{vw}}{2}\right]^{P}\sum_{k=0}^{p-1}C_{p-1+k}^{k}\left(\frac{1+\mu_{vw}}{2}\right)^{k}$$

where

$$C_n^m=\frac{n!}{m!\,(n-m)!}$$

and

$$\mu_{vw} = \sqrt{\frac{B\bar{\gamma}_{vw}}{2n_{tr}^{v}\, n_{rc}^{w} + B\bar{\gamma}_{vw}}} = \sqrt{\frac{B\bar{\gamma}_{vw}^{(mk)}}{B\bar{\gamma}_{vw}^{(mk)} + 2}}$$

with A=1, B=2 for the BPSK constellation and $\bar{\gamma}_{vw} = n_{tr}^{v}\, n_{rc}^{w}\, \bar{\gamma}_{vw}^{(mk)}$.

- P_{sr_k}, the BEP at the k-th relay, is given by:

$$P_{sr_k} = \theta(n_{tr}^{s}\, n_{rc}^{r_k}, s, r_k)$$

- P_{sd}, the BEP at the destination of the direct link, can be written as:

$$P_{sd} = \theta(n_{tr}^{s}\, n_{rc}^{d}, s, d).$$

IV. Simulation Results

In this section, the performance of OSTBC-MIMO system with incremental best relay selection using BPSK constellation is studied through simulations. As illustrated by the system model, the considered network consists of a source, M relays and a destination. We allocate the same power to the source and the activated relay, i.e. $P_x = {P_t}/{2}$, where P_t is the total system transmit power. We consider the path loss exponent is $\alpha = 3$.

We evaluate the performance of an incremental and incremental-selective DF MIMO system by plotting the curves of the bit error rate (BER) at the destination versus the SNR (dB). We compare the average BER of the studied system with different values of the threshold (γ_{th}). Fig. 2 illustrates the end to end average BER of the OSTBC-MIMO system with incremental relay selection (IRS) compared with the incremental relaying MIMO system (IR) and a cooperative relay selection MIMO system (RS). All the studied systems employ the DF protocol and the OSTBC for the 2111 ($n_{tr}^{s} = 2$, $n_{rc}^{r_k}$=1, $n_{tr}^{r_k}$=1, n_{rc}^{d}=1) scenario when M=2. The threshold, γ_{th}, is equal to 5 dB for IRS and IR systems. Fig. 2 demonstrates that incremental MIMO system with the best relay selection offers better performances in terms of BER than the same system with one relay. It can also be seen that RS system outperforms IRS and IR systems but it can cause a great loss in radio resources. As shown in Fig. 2, for a target $BER = 10^{-3}$, a $SNR = 9.5$ dB is required if the scheme RS is used, $SNR = 11$ dB is required for IRS and $SNR = 12$ dB for IR schemes. Thus, IRS have a gain of 1 dB compared to IR and have a loss of 1.5 dB compared to RS. If IRS has a higher BER compared to RS, it has a better spectral efficiency.

Fig. 3 shows the BER of OSTBC-MIMO system with DF incremental relay selection when γ_{th}=3 dB, γ_{th}=4 dB, γ_{th}=5 dB and M=2 for the 2111 ($n_{tr}^{s} = 2$, $n_{rc}^{r_k}$=1, $n_{tr}^{r_k}$=1, n_{rc}^{d}=1) scenario. Fig. 3 proves that if the threshold γ_{th} increases, the performances of the system improve.

Fig. 4 shows that the DF incremental selective MIMO system such as the 2111 scenario offers better performances in terms of BER than the incremental selective SISO one (1111: n_{tr}^{s}=1, $n_{rc}^{r_k}$=1, $n_{tr}^{r_k}$=1, n_{rc}^{d}=1).

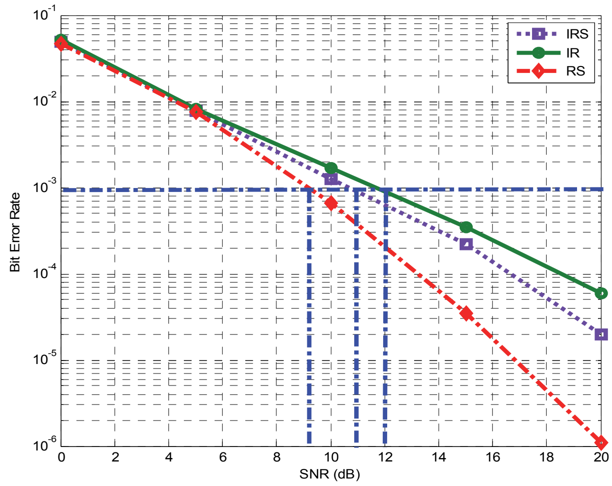

Fig. 2. BER comparison for the 2111 ($n_{tr}^{s} = 2$, $n_{rc}^{r_k} = 1$, $n_{tr}^{r_k} = 1$, $n_{rc}^{d} = 1$) scenario, γ_{th} = 5 dB and M=2.

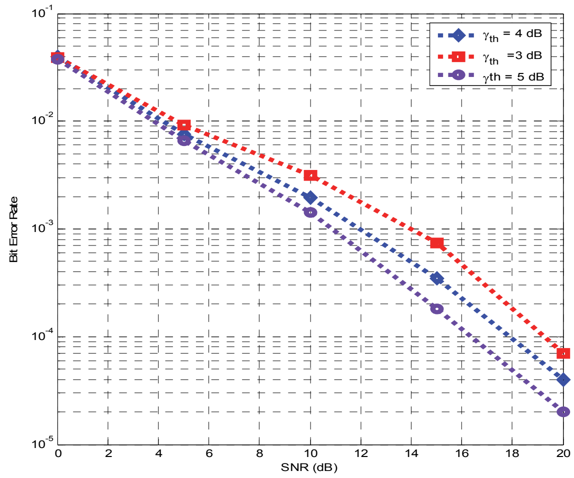

Fig. 3. BER performance of OSTBC-MIMO system with DF incremental relay selection for different values of γ_{th} for the 2111 scenario.

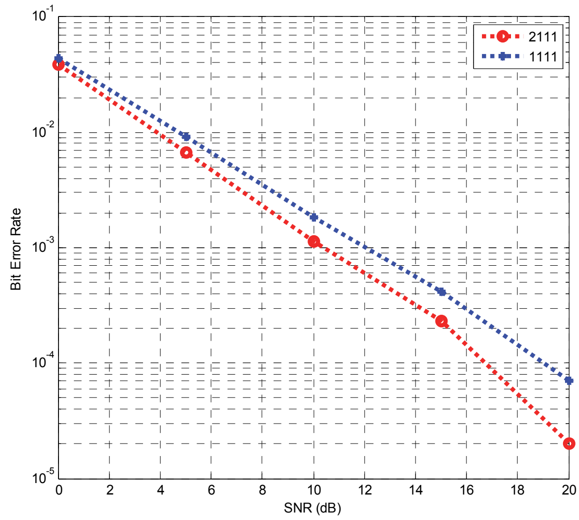

Fig. 4. BER performance of DF incremental selective MIMO relaying and SISO relaying for γ_{th}=5 dB, M=2.

V. Conclusion

In this paper, we have analyzed the end-to-end performance of the DF incremental OSTBC MIMO system with the best relay selection over Rayleigh fading channels. When the source requests relay assistance, the best relay among those that correctly decoded the received signal, is activated. We have derived an exact expression of the BEP at the destination, and the obtained results show that the incremental-selective OSTBC-MIMO system can improve the performance of the system compared to the case without relay selection.

Appendix

To calculate $\gamma_{d|r_{sel}}$, we follow these stages [15]:

$$\gamma_{d|r_{sel}} = \sum_{i=1}^{n_{rc}^d}\sum_{j=1}^{n_{tr}^s}\gamma_{sd}^{(ij)} + \sum_{k=1}^{n_{rc}^d}\gamma_{r_{sel}d}^{(k1)}$$
$$= \gamma_{\mathrm{sd}} + \gamma_{\mathrm{r_{sel}d}}$$

where

$$\gamma_{sd}^{(ij)} = \frac{E_s}{n_{tr}^s N_0}\left|h_{sd}^{(ij)}\right|^2 \text{ and } \gamma_{sd}^{(k1)} = \frac{E_{r_{sel}}}{n_{tr}^s N_0}\left|h_{r_{sel}d}^{(k1)}\right|^2.$$

$P_{d|r_{sel}}$, the BEP at the destination when the selected relay intervenes by sending a correctly decoded copy of the received signal, is given by:

$$P_{d|r_{sel}}=\int_0^\infty \frac{1}{2}\, erfc(\sqrt{\gamma})\; f_{\gamma_{d|r_{sel}}}(\gamma)\, d\gamma$$

where $f_{\gamma_{d|r_{sel}}}(\gamma)$ is the PDF of the received SNR at the destination when r_{sel} is the activated relay. To derive the expression of this PDF, we use the Moment Generating Function (MGF) of the SNR which can be written as:

$$M_{\gamma_{d|r_{sel}}}(s)=LT(f_{\gamma_{d|r_{sel}}}(\gamma))$$

where LT is the Laplace Transform. Thus, $M_{\gamma_{d|r_{sel}}}(s)$ can be expressed as:

$$M_{\gamma_{d|r_{sel}}}(s) = E(e^{-s\,\gamma_{d|r_{sel}}})$$
$$= \prod_{i=1}^{n_{rc}^d}\prod_{j=1}^{n_{tr}^s}\frac{1}{1+s\bar{\gamma}_{sd}^{(ij)}} + \prod_{k=1}^{n_{rc}^d}\frac{1}{1+s\bar{\gamma}_{r_{sel}d}^{(k1)}}$$

where

$$\bar{\gamma}_{sd}^{(ij)} = \frac{\bar{\gamma}_{sd}}{n_{tr}^s\, n_{rc}^d} \text{ and } \bar{\gamma}_{r_{sel}d}^{(k1)} = \frac{\bar{\gamma}_{r_{sel}d}}{n_{rc}^d}.$$

Thus

$$M_{\gamma_{d|r_{sel}}}(s) = \frac{1}{(1+s\frac{\bar{\gamma}_{sd}}{n_{tr}^s n_{rc}^d})^{n_{tr}^s n_{rc}^d}}\,\frac{1}{(1+s\frac{\bar{\gamma}_{r_{sel}d}}{n_{rc}^d})^{n_{rc}^d}}.$$

Using a fractional decomposition, we have:

$$M_{\gamma_{d|r_{sel}}}(s) = \sum_{l=1}^{n_{tr}^s n_{rc}^d}\frac{\delta_l}{(1+s\frac{\bar{\gamma}_{sd}}{n_{tr}^s n_{rc}^d})^l} + \sum_{k=1}^{n_{rc}^d}\frac{\lambda_k}{(1+s\frac{\bar{\gamma}_{r_{sel}d}}{n_{rc}^d})^k}$$

where δ_l and λ_k are the residues which were computed using the software Mathematica.

The PDF of the SNR can be calculated using the inverse Laplace Transform of the MGF as follows:

$$f_{\gamma_{d|r_{sel}}}(s) = LT^{-1}(M_{\gamma_{d|r_{sel}}}(s))$$
$$= \sum_{l=1}^{n_{tr}^s n_{rc}^d}\delta_l\varphi\left(x,\frac{\bar{\gamma}_{sd}}{n_{tr}^s n_{rc}^d},l\right) + \sum_{k=1}^{n_{rc}^d}\lambda_k\varphi\left(x,\frac{\bar{\bar{\gamma}}_{r_{sel}d}}{n_{rc}^d},k\right)$$

where $\varphi(\gamma, a, l)$ was previously defined in equation (3).

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