Frequency Offset Influence on MDPSK Signal Reception in Fading Channel

Slavimir N. Stošović, *Student Member, IEEE*, Bojan R. Dimitrijević, Nenad D. Milošević, and Zorica B. Nikolić, *Member, IEEE*

Abstract **— The symbol error probability of M-ary differential phase shift keying (MDPSK) receiver, in the presence of carrier frequency offset, is analysed in this paper. The paper proposes a novel multiple symbol differential detection (MSDD) receiver. The influence of various receiver's parameters on the error probability is analysed. The theoretical symbol error probability is derived and compared to the Monte-Carlo simulation results, for an AWGN channel. The analysis shows a good agreement between the theoretical and simulation results. The influence of the frequency offset in the Rician fading channel, using Monte-Carlo simulation, is also considered for the same receiver's parameters as in the AWGN channel. Simulation results show good system performances in the case of Rician fading channel, also.**

Keywords **— Adaptive signal processing; Differential phase shift keying; Frequency offset; Multiple-symbol differential detection, Rician fading channel.**

I. Introduction

It is well known that phase-shift keying (PSK) signals can be differentially or coherently demodulated. A coherent receiver, during operation, requires phase compliance with the transmitter, which is not easily solved. Differential detection uses the phase and frequency of the carrier corresponding to the previous transmitted data symbol as a demodulation reference. Hence, no phase tracking synchronization loop is necessary thus avoiding the difficulties of acquiring such a loop and maintaining it locked in a highly degraded environment. Nevertheless, coherent PSK has a slightly better performance than the differential PSK, but the coherent detection process is complicated because the synchronization of the receiver with the transmitter requires the knowledge of channel coefficients.

As indicated, a differential approach needs higher signal-to-noise power ratios (SNR) than coherent detection to achieve the same average bit error rate (BER) [1]. An attractive approach to mitigate this SNR loss is multiple-symbol differential detection (MSDD), which can be performed using a maximum likelihood sequence estimation [2], [3]. This detection technique is a general case of the conventional differential detection that uses more than two consecutive received samples to detect the information symbols. In [2] it is shown that by increasing the number of received samples in the MSDD algorithm, a receiver`s performance can be very similar to a coherent receiver performance, leading to large architectural complexity [4]. Because of this, several authors proposed algorithms with the same performance as MSDD, but with less complexity [5], [6].

The MSDD receiver proposed in [2] assumes that the frequency offset is equal to zero. In the case of nonzero frequency offset, the conventional MSDD technique must take the frequency offset into account; otherwise, when increasing the number of received samples which contribute in the receiver's metric, the performance of the receiver degrades very quickly [7,8]. In order to overcome this difficulty a double DPSK (DDPSK) modulation scheme has been proposed [9–11]. However, even under the most optimistic conditions, i.e. an infinite number of received samples and SNR $\rightarrow \infty$, the proposed receiver in [9] still needs a 3 dB higher SNR to achieve the same BER as the optimum coherent detector with DPSK modulation [9].

In this paper we propose a novel, modified MSDD receiver, which will be referred to as M-MSDD, for the reception of MDPSK signal. The influence of the frequency offset on the performance of the proposed receiver will be considered. There are both theoretical and simulation results. The results show good agreement between the theoretical and simulation results for the case of AWGN channel. Also, the results show good performance of the proposed algorithm in both AWGN and Rician fading channel.

II. System Model

A block diagram of the proposed MDPSK signal receiver used in this paper is shown in Fig. 1. Signal at the input of the receiver is:

$$r(t) = s(t) + n(t) \tag{1}$$

where $s(t)$ is the useful MDPSK signal:

$$s(t) = Ae^{j\theta(t)}e^{j\hat{\omega}_c t} \tag{2}$$

$$\theta(t) = \theta(t - T_s) + \frac{2\pi \cdot d(t)}{M} \tag{3}$$

This work was supported in part by the Ministry of Science and Technological Development of Serbia within the Project "Research and development of robust transmission systems for corporative networks" (TR-32037).

Slavimir Stošović is with The School of Higher Technical Professional Education, Aleksandra Medvedeva 20, 18000 Niš, Serbia, (phone: 381-69-4169060, e-mail: slavimir.stosovic@vtsnis.edu.rs).

Bojan Dimitrijević is with the Faculty of Electronic Engineering, University of Niš, Aleksandra Medvedeva 14, 18000 Niš, Serbia (phone: 381-18-529367; e mail: bojan.dimitrijevic@elfak.ni.ac.rs).

Corresponding Nenad Milošević is with the Faculty of Electronic Engineering, University of Niš, Aleksandra Medvedeva 14, 18000 Niš, Serbia (phone: 381-18-529367; e-mail: nenad.milosevic@elfak.ni.ac.rs).

Zorica Nikolić is with the Faculty of Electronic Engineering, University of Niš, Aleksandra Medvedeva 14, 18000 Niš, Serbia (phone: 381-18-529245; e-mail: zorica.nikolic@elfak.ni.ac.rs).

where symbol $d(t)$ has one of the following M values:

$$d(t) \in \{0,1,2,...,M-1\},\ pT_s \le t < (p+1)T_s,\ p = \pm 1, \pm 2... \quad (4)$$

T_s is the symbol interval, and $\hat{\omega}_c = \omega_c + \Delta\omega$ is the carrier frequency at the input of the receiver, ω_c is the locally generated fixed reference carrier frequency, $\Delta\omega$ is the frequency offset, and $n(t)$ is the white Gaussian noise with variance σ^2.

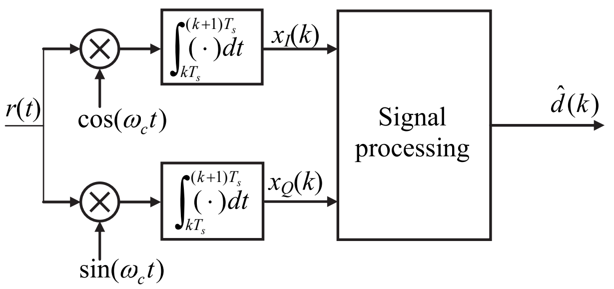

Fig. 1 Block diagram of the proposed M-DPSK signal receiver.

The input signal is multiplied by the fixed frequency reference carrier and passed through the integrate and dump circuit. The complex baseband signal at the input of the Signal processing block can be expressed as

$$X(k) = x_I(k) + jx_Q(k) \quad (5)$$

Signals at in-phase and quadrature branches are

$$x_I(k) = \int_{kT_s}^{(k+1)T_s} r(t)\cos(\omega_c t)dt$$
$$x_Q(k) = \int_{kT_s}^{(k+1)T_s} r(t)\sin(\omega_c t)dt \quad (6)$$

where k is a discrete time corresponding to the output of the integrate and dump circuit.

Signal estimation and detection is performed within the *Signal processing* block, and the estimated symbol $\hat{d}(k)$ is determined by the proposed M-MSDD algorithm that will be described in the following text. The proposed algorithm is based on the MSDD algorithm with the introduction of a mechanism for the estimation of the frequency offset. The algorithm is checking N_c frequency offsets around zero offset and chooses the one that is the most likely.

Based on [11, Eq. (9)], we set $N_H = 2^{N_S - 1}$ hypotheses using variable $R_i(k, n_h)$ as

$$R_i(k,n_h) = \mathrm{Re}\left\{ \sum_{l=1}^{L} \sum_{m=0}^{N_S-2} \sum_{n=m+1}^{N_S-1} X(k-m,l)X^*(k-n,l) \times \right.$$
$$\left. \times \exp\left(-j\left((m-n)\varepsilon_H(n_h) + \sum_{p=m}^{n-1} \theta_i(k,p) \right)\right) \right\} \quad (7)$$
$$i = 0,1,...,N_H - 1$$

where N_S is the number of symbols in multiple symbol detection,

$$\theta_i(k,p) = \left(\frac{i}{2^p} \bmod M \right)\pi \quad (8)$$

$$\varepsilon_H(n_h) = n_h\varepsilon_s - \frac{(N_H - 1)\varepsilon_s}{2},\ n_h = 0,...,N_H - 1 \quad (9)$$

where N_H is the number of channels that M-MSDD algorithm is operating with, and ε_s is the algorithm parameter that represents a phase step. The idea is to try to detect a symbol with the assumption that the frequency offset is equal to $-(N_H - 1)\varepsilon_s / 2T_S$, $-(N_c - 3)\varphi_s / 2T$, …, $(N_H - 3)\varepsilon_s / 2T_S$, $(N_H - 1)\varepsilon_s / 2T_S$. The best assumption (out of given N_c) will give the estimated frequency offset. In ordinary MSDD algorithm, N_H would be equal to $N_H = 1$, and $\varepsilon_H(n_h) = 0$.

Now, we first find the maximum value of $R_i(k,n_h)$ with respect to i

$$R_{\max}(k,n_h) = \max_i R_i(k,n_h) \quad (10)$$

To mitigate the effect of noise, $S(k,n_h)$ keeps low pass filtered $R_{max}(k, n_h)$ values:

$$S(k,n_h) = S(k-1,n_h)(1-A) + A \cdot R_{\max}(k,n_h) \quad (11)$$

where A is the filter parameter. The next step is to find n_h that maximizes $S(k,n_h)$:

$$n_{h\max}(k) = \arg\max_c S(k,n_h) \quad (12)$$

Using $n_{h\max}(k)$ it is possible to find i that maximizes $R_i(k,n_h)$

$$i_{\max}(k) = \arg\max_i R_i(k, n_{h\max}(k)) \quad (13)$$

The detected symbol is equal to

$$\hat{d}(k) = \frac{i_{\max}(k)}{2^{\frac{N_B - 1}{2}}} \bmod 2 \quad (14)$$

III. Theoretical Analysis

The proposed receiver estimates the carrier frequency offset. Therefore, after the estimation, the frequency offset is changed and likely smaller than the original frequency offset ε. The effective frequency offset after the estimation will be referred to as ε_{eff}.

Considering that the low frequency filter is applied (11), detection variable averaging is performed for each hypothesis. Thus, we can consider that detection is sufficiently reliable. The optimal n_h, n_h' ,is determined as:

$$n_h' = \arg\min_{n_h}(\varepsilon - \varepsilon_H(n_h)) \quad (15)$$

The effective frequency offset is equal to

$$\varepsilon_{eff} = \varepsilon - \varepsilon_H(n_h') \quad (16)$$

The BER for the binary differential phase shift keying (BDPSK) modulation is defined with the following equation

$$P_e = \frac{1}{2} e^{-\cos^2(2\pi\varepsilon_{eff})SNR} \quad (17)$$

where SNR represents a signal to noise power ratio given as

$$SNR = \frac{A_S^2}{\sigma^2} \quad (18)$$

and A_S is the signal amplitude.

For M-ary differential phase shift keying, where modulation level $M \ge 4$, based on [12, Eq. (25)] SER approximation formula is given with

$$P_e = Q\left(\sqrt{2\sin^2\left(\frac{\pi}{M} + 2\pi\varepsilon_{eff}\right)SNR'}\right) + Q\left(\sqrt{2\sin^2\left(\frac{\pi}{M} - 2\pi\varepsilon_{eff}\right)SNR'}\right) \quad (19)$$

where $Q(x)$ is the Gaussian Q-function, and SNR' represents the equivalent signal to noise ratio for differential detection. It can be shown that

$$SNR' = \frac{SNR^2}{2SNR+1}, \quad (20)$$

IV. Numerical Results

The results shown in this section are obtained by the Monte-Carlo simulation with $2\cdot10^6$ symbols. Also, there are theoretical results, based on the previous analysis. System parameters are: carrier frequency $f_c = 2400$ MHz, symbol rate $1 / T_S = 100$ kSym/s.

Fig. 2 shows the influence of the number of symbols (N_S) in M-MSDD algorithm on the symbol error probability, for a zero frequency offset, in the AWGN channel. There are three groups of curves for BDPSK, 4DPSK and 8DPSK modulation format. It may be noticed that N_S has a much higher influence on SER for 4DPSK and 8DPSK modulations than for BDPSK. Also, when N_S is high, increasing the number of hypotheses N_H has no effect on system performances. There is also good agreement between theoretical and simulation results, except for 4DPSK. This is so because approximation (19) stands for $M \geq 4$, and is the least accurate for $M = 4$.

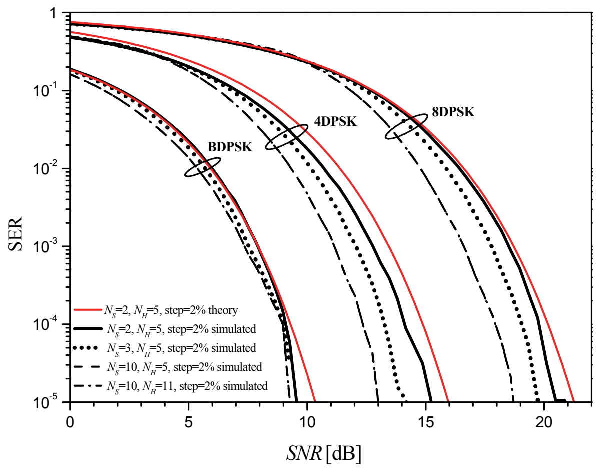

Fig. 2. Symbol error probability as a function of E_b/N_0 for $\Delta f = 0$ Hz and MBDD.

Symbol error probability as a function of frequency offset for BDPSK modulation is shown in Fig. 3. With increasing of parameter N_S, the error probability error decreases for a zero frequency offset, but the system is more sensitive to the frequency offset. This is in accordance with the nature of MSDD algorithm. By increasing the number of hypotheses, the system becomes more resistant to the frequency offset. Again, as in Fig. 2, there is a good agreement between theoretical and simulation results.

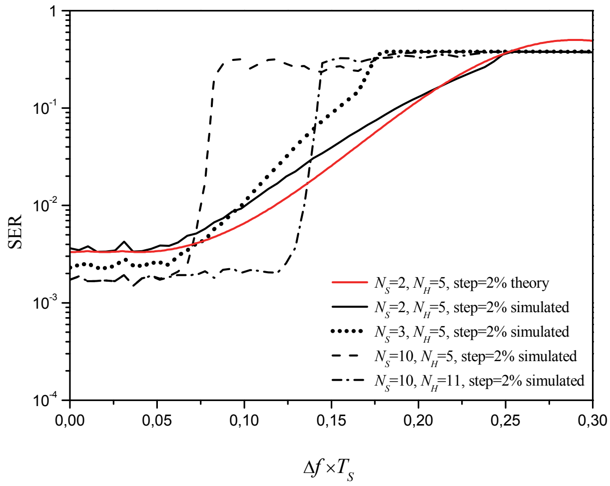

Fig. 3. Symbol error probability as a function of frequency offset for BDPSK modulation in AWGN channel, $SNR = 7$dB.

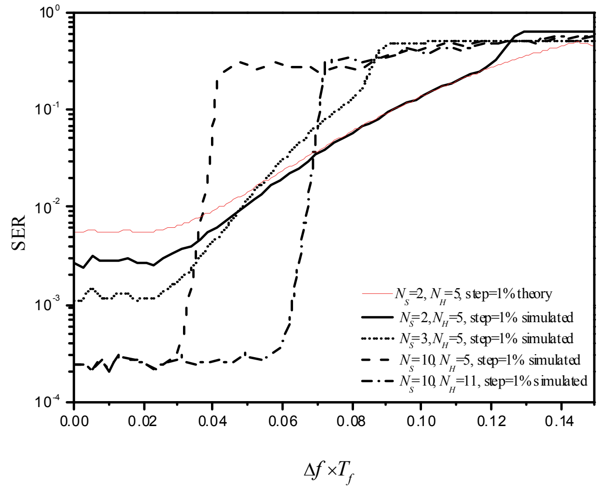

Fig. 4. Symbol error probability as a function of frequency offset for 4DPSK modulation in AWGN channel, $SNR = 12$dB.

Figs. 4 and 5 present the symbol error probability of the proposed system for 4DPSK and 8DPSK modulation, respectively. The conclusions are similar as for Fig. 3, except for a slight disagreement between theoretical and simulation results for 4DPSK, because of the already mentioned reasons.

Since the presented scheme is intended for the mobile and wireless systems where a fading channel is assumed, in the following figures the performance analysis is performed in the Rician fading channel with the Rician factor $K = 10$ dB. The values of SNR, for BDPSK, 4DPSK and 8DPSK modulation, are the same as in the AWGN channel.

Symbol error probability is shown in Fig. 6 as a function of frequency offset for BDPSK modulation in the Rician fading channel. As in the case of AWGN channel, with the increase of parameter N_S the error probability decreases for a zero frequency offset. However, for a larger N_S, the system is more sensitive to the frequency offset. This sensitivity may be reduced by increasing the number of hypotheses.

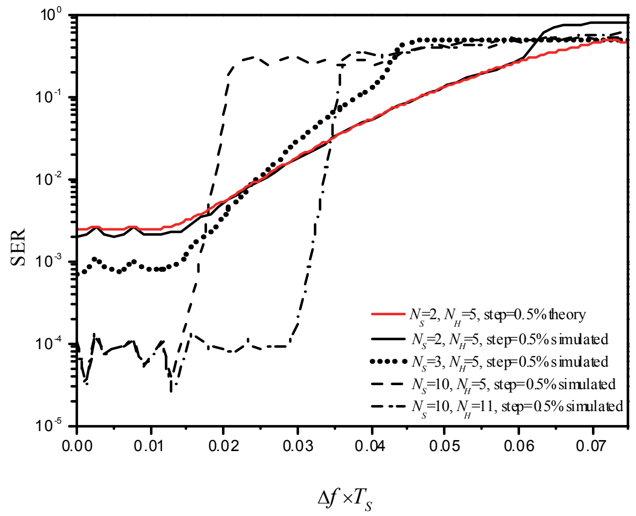

Fig. 5. Symbol error probability as a function of frequency offset for 8DPSK modulation in AWGN channel, *SNR* = 18dB.

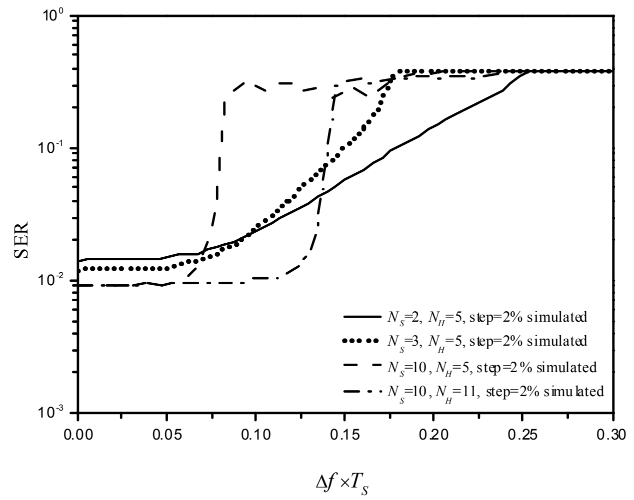

Fig. 6. Symbol error probability as a function of frequency offset for BDPSK modulation in Rician fading channel, *SNR* = 7dB.

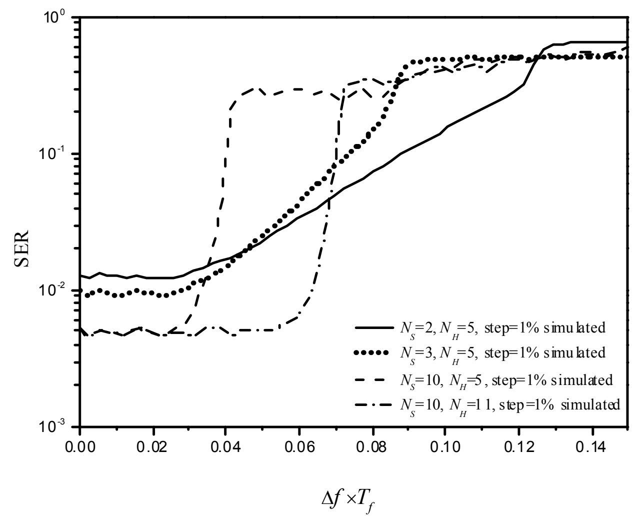

Fig. 7. Symbol error probability as a function of frequency offset for 4DPSK modulation in Rician fading channel, *SNR* = 12dB.

Fig. 7 presents the symbol error probability of the proposed system in the Rician fading channel for 4DPSK modulation. The conclusions are similar as for Fig. 6.

V. Conclusion

In this paper we have proposed a modified MSDD receiver for the reception of MDPSK signal. The influence of the frequency offset on the performance of the proposed receiver, in the AWGN channel and Rician fading channel, is considered using Monte-Carlo simulation. For both channels, the analysis in case of zero frequency offset shows that N_S has a much higher influence on SER for 4DPSK and 8DPSK modulations than for BDPSK, and when N_S is high, the increase of the number of hypotheses N_H has no effect on the system performance. With the increase of parameter N_S, error probability decreases for a zero frequency offset, but the system is more sensitive to the frequency offset. By increasing the number of hypotheses, the system becomes more resistant to the frequency offset. There is also good agreement between theoretical and simulation results for the AWGN channel.

References

[1] J. G. Proakis, "Digital Communications," 4th ed. New York: McGraw-Hill, 2000.

[2] D. Divsalar, M. K. Simon, "Multiple-symbol differential detection of MPSK," *IEEE Transactions on Communications*, vol. *38*(3), pp. 300–308, 1990.

[3] C. Gao, A. M. Haimovich, D. Lao, "Multiple-symbol differential detection for MPSK space-time block codes: decision metric and performance analysis," *IEEE Transactions on Communications*, vol. *54*(8), pp. 1502–1510, 2006.

[4] N. Hamamoto, "Differential detection with IIR filter for improving DPSK detection performance," *IEEE Transactions on Communications*, vol. *44*(8), pp. 959–966, 1996.

[5] K. M. Mackenthun Jr, "A fast algorithm for multiple-symbol differential detection of MPSK," *IEEE Transactions on Communications*, vol. *42*(2/3/4), pp. 1471–1474, 1994.

[6] G. Colavolpe, R. Raheli, "Noncoherent sequence detection," *IEEE Transactions on Communications*, vol. *47*(9), 1376–1385, 1999.

[7] B. Dimitrijević, S. Stošović, N. Milošević, Z. Nikolić, "Performances of the OFDM/QPSK system with MBDD in the presence of frequency offset," In *Proceedings of the TELFOR 2010* pp. 475–478, Belgrade, Serbia, Nov. 2010.

[8] M. K. Simon, D. Divsalar, "On the implementation and performance of single and double differential detection schemes," *IEEE Transaction on Communications*, vol. *40*(2), 278–291, D. 1992.

[9] D. Divsalar, M. K. Simon, "Double differential detection," *NASA New Technology Item 7170*, Docket 17666, June 27, presented at *IEEE Communications Theory Workshop*, Sedona, AZ, Apr. 1988.

[10] Yu. B. Okunev, V. A. Pisarev, V. K. Reshemkin, "The design and noise immunity of multiphase autocorrelation demodulators of second order DPSK signals," *Radiotekhnika*, vol. *34*(6), pp. 60–63, 1979.

[11] A. M. Rabiei, N. C. Beaulieu, "Multiple symbol differential detection of MPSK in the presence of frequency offset," In *Proceedings of the IEEE International Conference on Communications*, pp. 693–697, Seoul, Korea, May. 2005.

[12] J. Lee, D. Toumpakaris, H.-L. Lou, J. M. Cioffi, "Effect of carrier frequency offset on time-domain differential demodulation in OFDM systems," In *Proceedings of Vehicular Technology Conference*. vol. I, pp. 568 – 572, Sept. 2004.