Approximately Liner Phase IIR Digital Filter Banks

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Abstract — **In this paper, uniform and nonuniform digital filter banks based on approximately linear phase IIR filters and frequency response masking technique (FRM) are presented. Both filter banks are realized as a connection of an interpolated half-band approximately linear phase IIR filter as a first stage of the FRM design and an appropriate number of masking filters. The masking filters are half-band IIR filters with an approximately linear phase. The resulting IIR filter banks are compared with linear-phase FIR filter banks exhibiting similar magnitude responses. The effects of coefficient quantization are analyzed.**

Keywords — **approximately linear phase IIR filters, FRM, non-uniform filter bank, uniform filter bank.**

I. INTRODUCTION

UNIFORM and nonuniform linear-phase nonsubsampled filter banks are a common choice in real-time processing of audio signals [1], [2]. In the case of selective filter banks, directly designed linear phase FIR filters of a very high order are requested. The computational complexity and the overall delay of the filter bank become intolerably high. Filter design based on frequency response masking (FRM) technique can significantly reduce the computational complexity. The FRM techniques are used in the design of very selective narrowband and wideband FIR [3] and IIR [4] filters. In both cases, filtering is realized by cascading two or several low order filters: the first stage is an interpolated prototype FIR or IIR filter, and the second stage consists of one or more linear phase (masking) FIR filters. If the first stage filter is an FIR filter, the computational complexity of the overall filter composed of interpolated filter and masking filters is smaller compared to the straightforward design (without FRM). If the first stage filter is an IIR filter, design based on the FRM technique is less sensitive to the finite world length effects compared to the IIR filter realized in a conventional way. Additionally, the use of an IIR interpolation filter instead of the FIR filter decreases the overall delay of the FRM-based overall filter.

In real time audio applications, both high selectivity and low delay are needed. An efficient design of uniform filter bank for audio systems is presented in [1]. Solution for the design of nonuniform filter bank suitable for the hearing-aid devices is described in [2]. In both cases filter banks are designed by means of FRM technique. Prototype filters are low order linear-phase FIR half-band filters. By interpolation of the prototype filter, a multichannel filter is achieved. In the next stage(s) channels are separated by one or several masking filters. Masking filters are also linear-phase half-band FIR filters. It should be noted that all filters involved in the design of uniform filter bank presented in [1] and nonuniform filter bank presented in [2] are half-band filters (or interpolated half-band filters). Therefore, it seems promising that half-band IIR filters with an approximately linear phase [5] can be used instead of FIR filters. It is expected that in that way the overall delay and filter bank computational complexity can be decreased.

FRM technique was used for the design of IIR filters [4]. The solution presented in [4] is based on the prototype IIR filter realized as a parallel connection of two allpass filters and linear-phase FIR masking filters. In our approach, all filters are approximately linear-phase IIR filters. This is possible because all filters in the bank are half-band filters.

The approximately linear-phase IIR filter pair used as a building block in the uniform and nonuniform filter banks of this paper is a special case of filter pair realized as a parallel connection of two all-pass branches. For that reason, it can be assumed that the passband sensitivity is small and that the stopband sensitivity is large [6]. For all filter pairs realized as a parallel connection of all-pass filters achieving desired stopband attenuation is a critical task. Therefore, we analyzed coefficients quantization effects for proposed structures.

This paper is organized as follows: in section 2 the filter bank structures for the proposed uniform and nonuniform filter banks are explained, in section 3 the results of the comparison with filter banks from [1] and [2] are presented, in section 4 fixed point realizations for both filter banks are discussed, and section 5 concludes the paper.

II. FILTER BANKS STRUCTURES

Based on the realization structures presented in [1] and [2] filter banks with an approximately linear phase with reduced overall delay compared to filter banks [1] and [2] are developed. The basic element for both filter banks is a complementary IIR filter pair [$H(z)$ $H_c(z)$] with an approximately linear phase, defined as:

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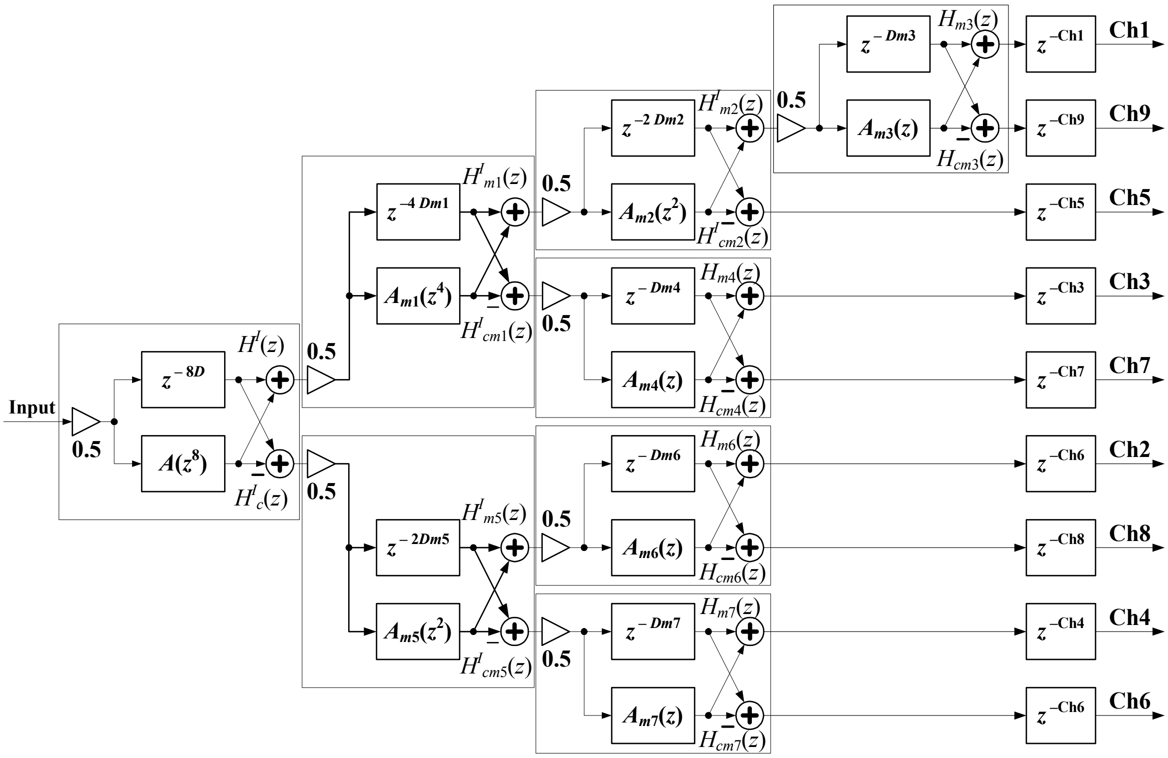

Fig. 1. Nine-channel uniform filter bank based on approximately linear phase IIR filters.

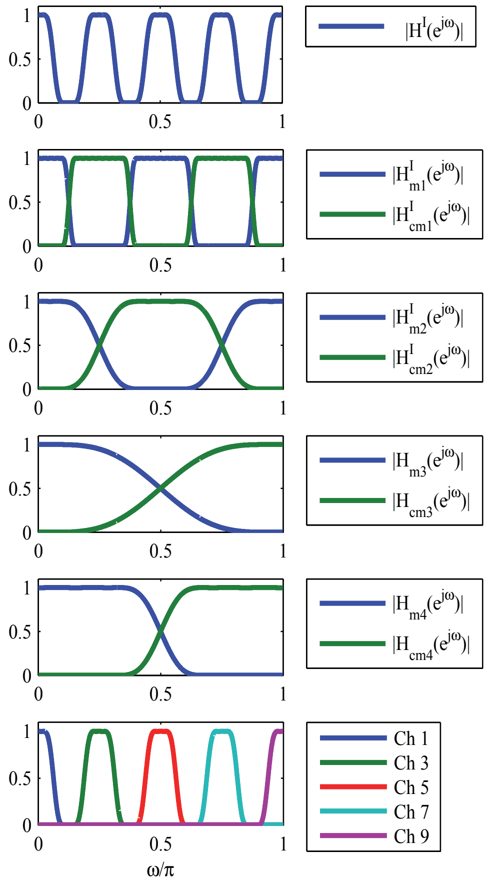

Fig. 2 Realization of odd channels of the uniform filter bank.

$$H(z)=\frac{z^{-D}+A(z)}{2},\qquad H_c(z)=\frac{z^{-D}-A(z)}{2}, \tag{1}$$

where D is an integer, $A(z)$ is the all-pass filter of order D+1 designed by the optimization procedure [5]. The passband group delay of the filter pair [$H(z)$ $H_c(z)$] amounts to approximately D samples. Filter pair [$H(z)$ $H_c(z)$] exhibits delay-complementary and power-complementary properties [7].

A. Uniform Filter Bank

The uniform filter bank is composed as a connection of an interpolated prototype filter $H(z)$ and several masking filters $H_{mi}(z)$ [1]. We modified the structure proposed in [1] by replacing the half-band FIR filters by approximately linear phase half-band IIR filters (1).

An IIR 9-channel filter bank is presented in Fig. 1. By interpolation of the prototype filter $H(z)$ multiband filter $H^I(z)$ is obtained. Each passband represents one of the five odd channels of the resulting filter bank. Even channels are obtained by interpolation of the complementary filter $H_c(z)$. Subbands of the filter bank are separated from multichannel filters $H^I(z)$ and $H^I_c(z)$ by appropriate connection of masking filters [1]. Masking filters are either half-band filters or interpolated half-band filters. Additional delays are introduced in all channels (except for the channel with the longest delay). These additional delays guarantee that the realized filter bank will retain double complementary (delay-complementary and power-complementary) properties. The amplitude responses of interpolated prototype filter $H^I(z)$ and masking filters of odd channel branches are sketched in Fig. 2. Even channels responses are obtained in a similar manner,

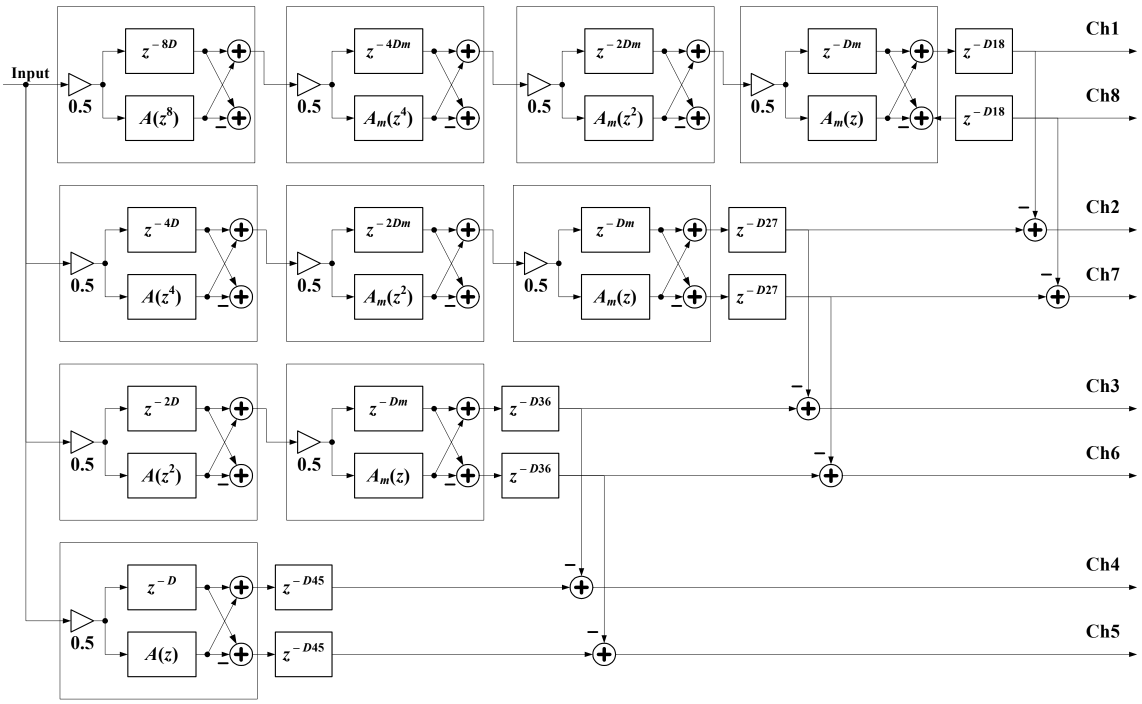

Fig. 3. Eight-channel nonuniform filter bank based on approximately linear phase IIR filters.

starting with interpolated complementary prototype filter $H^I_c(z)$. In Table 1 passband edge frequencies and all-pass branch orders of all prototype (noninterpolated) filters are given.

All filters are designed as half-band approximately linear phase filters with 60 dB stopband attenuation. The number of multiplications needed for the calculation of a single output sample for each filter is calculated under the assumption that the all-pass branch of IIR filter is realized as a cascaded connection of the second- and fourth-order sections. The fourth-order sections are half-band (even coefficients are zero valued) filters. Therefore, only two multiplications are needed for calculation of the filter output [7].

Based on the design procedure described above, filter banks with a different number of channels can be realized. As can be seen from Table 1, the masking filter $H_{m7}(z)$ is of the highest order. The masking filter $H_{m7}(z)$ separates even channels (4 and 6) that are closest to the frequency 0.5π, thus should be very selective. For all filter banks based on the proposed design (FIR or IIR with a different number of channels) even bands that are closest to the frequency 0.5π are critical. Overall delay of the filter bank depends on the delay of that critical masking filter. If the number of channels increases, the requirements for the selectivity of critical masking filter increase as well. Therefore, delay of that masking filter and overall delay of the filter bank can be intolerably long in the case of filter bank with an increased number of channels [8].

B. *Nonuniform Filter Bank*

In reference [2], an efficient realization of 8-channel nonuniform linear phase filter bank suitable for hearing-aid applications is presented. We modified the solution presented in [2] by replacing all linear phase half-band FIR filters with approximately linear phase half-band IIR filters. Each channel of the IIR filter bank consists of a connection of interpolated prototype filter $H(z)$ and one or a number of masking filter(s) $H_m(z)$. Different channels, i.e., different passband frequencies, are obtained by interpolation of prototype filter $H(z)$ by a corresponding factor [2]. The cascaded connection of interpolated and noninterpolated masking filter(s), remove unwanted passbands of the interpolated prototype filter, Fig. 3. The entire filter bank is based on only two different filters [2]. In Table 2 the parameters of prototype filters are given. Both filters are designed as approximately linear phase IIR filters [5], with the stopband attenuation of 80 dB. Channels of the nonuniform filter bank are determined by the 3 dB-crossover frequencies: 0.0625π, 0.125π, 0.25π, 0.5π, 0.75π, 0.875π and 0.9375π [2]. The gain responses of all channels of the filter bank are presented in Fig. 4.

III. EFFICIENCY ANALYSIS OF IIR FILTER BANKS

IIR filter banks proposed in this paper are analyzed and compared with FIR filter banks presented in [1] and [2].

A. *Uniform Filter Bank*

Table 3 displays the filter orders of FIR filter bank exhibiting a similar amplitude response to that of the IIR filter bank presented in section II. The overall delay depends on the delay of channels 4 and 6:

$$TD = 8D_H + 2D_{m5} + D_{m7} \qquad (2)$$

where TD is a total delay, D_H is delay of the prototype filter $H(z)$ and D_{mi} is delay of the i-th masking filter. For the FIR filter bank, delay of each filter is:

$$D_{FIR} = 0.5N_{FIR}, \qquad (3)$$

where N_{FIR} is the filter order. For the IIR filter bank, delay of the individual filters can be calculated by:

$$D_{IIR} = N_{IIR} - 1, \qquad (4)$$

where N_{IIR} is the all-pass branch order.

TABLE 1: UNIFORM IIR FILTER BANK PROTOTYPE FILTER PARAMETERS.

Filter	*All-pass branch order*	*Number of multiplications*	*Passband edge frequency*
$H(z)$	4	2	0.2π
$H_{m1}(z)$	14	7	0.4π
$H_{m2}(z)$	4	2	0.2π
$H_{m3}(z)$	4	2	0.1π
$H_{m4}(z)$	10	5	0.35π
$H_{m5}(z)$	26	13	0.45π
$H_{m6}(z)$	4	2	0.225π
$H_{m7}(z)$	48	24	0.475π

TABLE 2: NONUNIFORM IIR FILTER BANK PROTOTYPE FILTER PARAMETERS.

Filter	*All-pass branch order*	*Number of multiplications*	*Passband edge frequency*
$H(z)$	8	4	0.25π
$H_m(z)$	16	8	0.375π

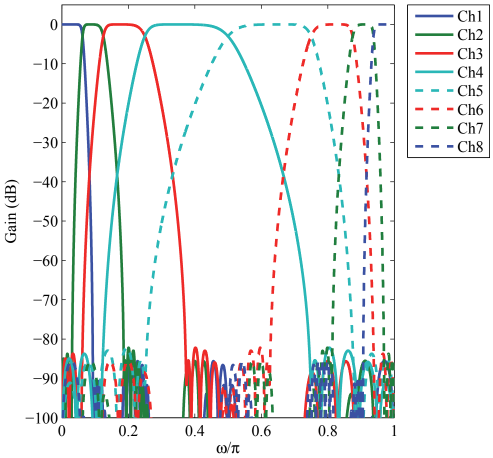

Fig. 4. Nonuniform filter bank gain responses.

For the FIR filter case the total delay is 181 samples and for the IIR case the total delay is 121 samples. This result is verified by calculating response to the square input signal, Fig. 5. Table 4 gives a comparison of FIR and IIR uniform filter banks.

B. *Nonuniform Filter Bank*

In the case of the IIR filter bank the overall delay equals the delay of the first (and eighth) channel:

$$TD = 8D_H + 4D_m + 2D_m + D_m, \qquad (5)$$

where TD is a total delay, D_H is delay of the prototype filter $H(z)$ and D_m is delay of the noninterpolated masking filter.

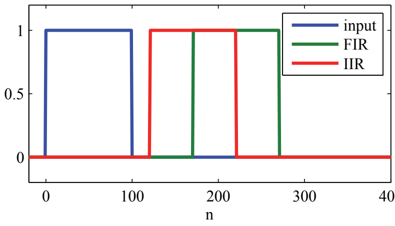

Fig. 5. Uniform filter bank square input responses.

TABLE 3: UNIFORM FIR FILTER BANK PROTOTYPE FILTER PARAMETERS.

Filter	*Filter order*	*Number of multiplications*
$H(z)$	10	4
$H_{m1}(z)$	34	10
$H_{m2}(z)$	10	4
$H_{m3}(z)$	6	3
$H_{m4}(z)$	22	7
$H_{m5}(z)$	66	18
$H_{m6}(z)$	10	4
$H_{m7}(z)$	130	34

TABLE 4: UNIFORM FILTER BANK – COMPARISON OF FIR AND IIR REALIZATIONS.

Filter	*Delay (samples)*	*Number of multiplications*	*Complementary*
FIR	181	79	delay
IIR	121	57	delay, power

TABLE 5: NONUNIFORM FIR FILTER BANK PROTOTYPE FILTER PARAMETERS.

Filter	*Filter order*	*Number of multiplications*
$H(z)$	18	6
$H_m(z)$	38	11

For the FIR filter case the total delay is 205 samples ($H(z)$ is of order 18, $H_m(z)$ is of order 38, see Table 5) and for the IIR filter case the total delay is reduced to 161 samples, see Fig. 6. Table 7 presents the results of comparison of FIR and IIR realizations. Data for the number of multiplications are derived based on the hardware sharing proposed in [2].

TABLE 6: NONNIFORM FILTER BANK – COMPARISON OF FIR AND IIR REALIZATIONS.

Filter	*Delay (samples)*	*Number of multiplications*	*Complementary*
FIR	205	17	Delay
IIR	161	12	delay, power

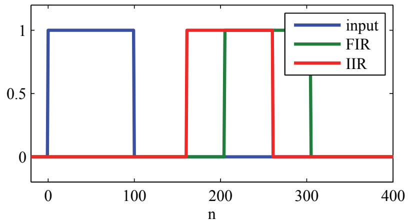

Fig. 6. Nonuniform filter bank square input responses.

IV. FIXED-POINT REALIZATION

Approximately linear phase IIR filters are implemented as a parallel connection of an all-pass branch and a pure delay, Fig. 7. Allpass filter $A(z)$ is of order $4k$ or $4k+2$. In the 4k+2 case, allpass filter $A(z)$ has two poles on the imaginary axes and k quadruplets of poles, Fig. 8:

$$z_{4k,4k+1,4k+2,4k+3} = \pm x_k \pm i y_k, \; i = \sqrt{-1}. \qquad (6)$$

In the $4k$ case, there is an additional pair of poles placed on the real axes, Fig. 9.

Each quadruplet of poles with corresponding zeros forms a single fourth order all-pass section:

$$A_k(z)=\frac{a_{k4}+a_{k3}z^{-1}+a_{k2}z^{-2}+a_{k1}z^{-3}+z^{-4}}{1+a_{k1}z^{-1}+a_{k2}z^{-2}+a_{k3}z^{-3}+a_{k4}z^{-4}}. \quad (7)$$

Since $a_{k1}=a_{k3}=0$, $A_k(z)$ can be written in the form:

$$A_k(z)=\frac{\beta_k+\alpha_k(1+\beta_k)z^{-2}+z^{-4}}{1+\alpha_k(1+\beta_k)z^{-2}+\beta_k z^{-4}}, \quad (8)$$

where $\beta_k=\left(x_k^2+y_k^2\right)^2$. The structure of all-pass fourth-order section given in (8) is suitable for the implementation in the form of Ansari-Liu section [9]. Any of the second-order section types proposed in [9] can be used if each delay is replaced with two delays. Implemented in this way, each fourth-order section needs only two multiplications.

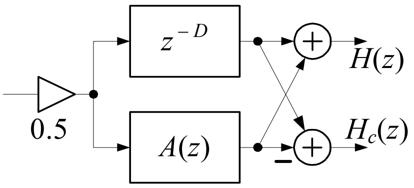

Fig. 7. Realization structure of approximately linear phase filter pair [$H(z)$ $H_c(z)$].

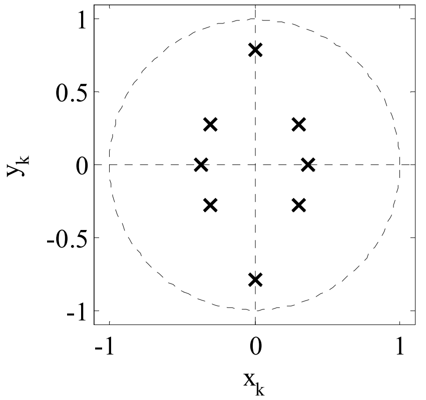

Fig. 8. Poles of the allpass branch of order $4k$+2.

In a similar manner, second-order sections implementing the imaginary axes pole pairs (and real axes pole pairs if the filter order is $4k$) can be reduced to:

$$A_0(z)=\frac{\gamma+z^{-2}}{1+\gamma z^{-2}}, \quad (9)$$

and realized as a modified first-order Ansari-Liu section [9]. Any form of the first order sections presented in [9] can be used, and, again, a delay is replaced by two delays. As a result, only one multiplication is needed for the realization of the second-order section presented by (9).

Interpolation of the filters (Figs 1 and 3) is performed by replacing each delay element with a number of delay elements that equals the interpolation factor. As a result, the fourth-order section is transformed into the $4L$-order section, where L is the interpolation factor. Poles of the interpolated section (Fig. 9, poles colored in red) for L = 4 are presented in Fig. 10. However, computation of the output of the interpolated section requires the same number of multiplications, i.e., two for the starting section of the order four, and only one for the starting section of the order two.

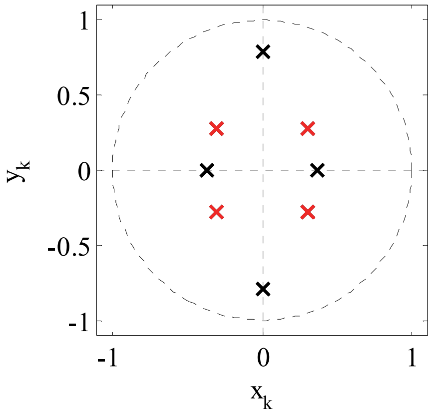

Fig. 9. Poles of the allpass branch of order $4k$.

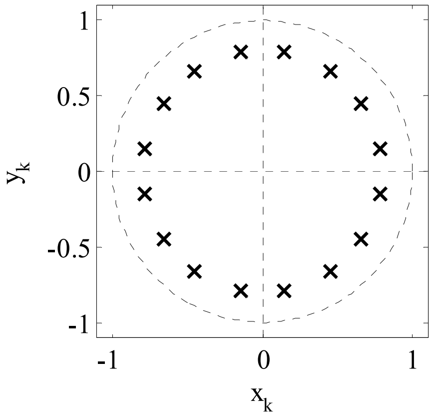

Fig. 10. Poles of the interpolated fourth-order allpass section for L = 4.

Fixed-point realization is simulated in MATLAB. Frequency responses of uniform and nonuniform filter banks are calculated assuming that all filters are implemented with coefficients α_k, β_k and γ represented as 16 bit two's complement numbers. In the case of uniform filter bank, overall design consists of eight different filters. It can be assumed that not all filters require 16 bit word-length coefficients. We repeated our simulation with reduced wordlengths, and with different wordlength values for the second-order (real axes pole pair - RA and imaginary axes pole pair - IA) and fourth-order sections. Our simulation proved that achieving a required stopband attenuation is a critical task, as it is expected for all IIR filters realized as a parallel connection of two all-pass filters [6]. Filter coefficient wordlengths for which frequency responses satisfy design specifications are given in Table 7.

As it was expected, filter $H_{m7}(z)$ requires the longest wordlengths. For the filter $H_{m7}(z)$ order of 48 (Table 1) in fixed-point implementation required more than 16 bits. Therefore, the filter order had to be increased by two (from 48 to 50).

Group delay (GD) and gain responses of all the channels of the uniform filter bank are presented in Fig 11. It can be seen that the obtained group delay is flat in the passband of each channel. Group delay of the entire filter bank (sum of all the channels) is presented as a dashed black line. As expected because of the all-pass complementarity of the proposed structure, the overall group delay is ideally flat. Gain response details of the passbands and of the stopbands are shown. Results obtained with wordlengths given in Table 7 (red lines) are compared to the responses calculated without filter coefficients quantization (black lines). It can be seen that the stopband attenuation of 60 dB is achieved with coefficients quantized according to Table 7.

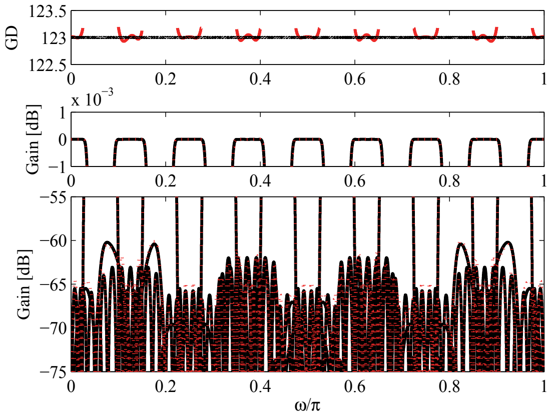

Fig. 11. Uniform filter bank response.

TABLE 7: UNIFORM IIR FILTER BANK COEFFICIENTS WORD-LENGTHS.

Filter	***Second Order (IA)***	***Second Order (RA)***	***Fourth Order***
$H(z)$	10	7	-
$H_{m1}(z)$	11	-	13
$H_{m2}(z)$	10	7	-
$H_{m3}(z)$	11	11	-
$H_{m4}(z)$	12	-	13
$H_{m5}(z)$	13	-	15
$H_{m6}(z)$	12	11	-
$H_{m7}(z)$	14	-	15

Similar simulation was performed for the nonuniform filter bank. In the case of nonuniform filter bank, there are only two different filters. Coefficients wordlengths are given in Table 8.

Group delay and a comparison of the frequency responses obtained with (red lines) and without (black lines) coefficient quantization are presented in Fig. 12. Group delays of the channels approximate flat characteristics in the corresponding passbands. However, the overall group delay (dashed black line) is ideally flat. Gain response passband and stopband details are shown. As can be seen, the stopband attenuation of 80 dB is achieved with coefficients quantized according to Table 8.

TABLE 8: NONUNIFORM IIR FILTER BANK COEFFICIENTS WORD-LENGTHS.

Filter	***Second Order (IA)***	***Second Order (RA)***	***Fourth Order***
$H(z)$	12	12	13
$H_m(z)$	14	14	15

V. CONCLUSION

Filter banks based on the IIR approximately linear phase half-band filters are more efficient compared to the FIR filter banks with similar amplitude responses. The overall delay of the IIR filter banks is smaller than the delay of the FIR filter banks. IIR filter banks are delay-complementary and power-complementary, while FIR filter banks are delay-complementary only. An ideally linear phase response and a constant group delay can be achieved by FIR filter banks. In the case of IIR filter banks, phase responses have an approximately linear characteristic in the channels' passbands. Both filter banks can be implemented in fixed-point arithmetic with maximum required wordlength of 16 bits. Further improvements can be achieved by appropriate realization of the IIR filters, for example, as multiplierless structures [10].

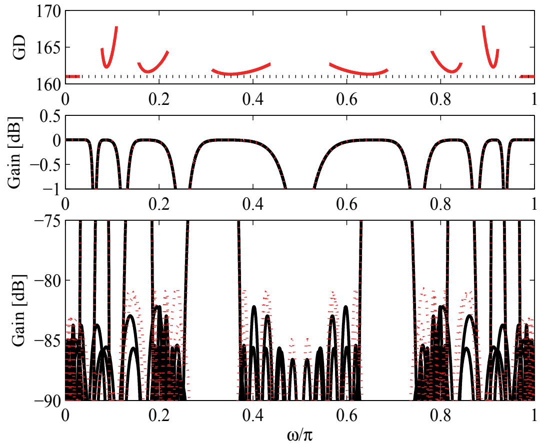

Fig. 12. Nonuniform filter bank response.

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