Non-Stationary Two-Dimensional Subband Transformer Filters

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***Abstract* — State-of-the-art two-dimensional subband transformation based compression methods typically require large memory size and memory bus bandwidth. This paper discloses a novel method for reducing both shortcomings, and even provides compression or decompression from the cache memory. This solution can be implemented either in software or hardware or their combination, as a front stage in either lossless or lossy image encoder or a back stage in either lossless or lossy image decoder.**

Keywords — Image compression, subband transformation, non-stationary filters.

I. Introduction

SUBBAND transformation coefficients are typically computed by first filtering and downsampling uncompressed input pixels and then subsequently filtering and downsampling resulted intermediate transformation coefficients with a set of lowpass and highpass filters. Each subband is separately coded with a bit rate that matches the importance of the subband. This leads to pleasing image reconstruction and minimization of compression artifacts.

The concept of subband transformation was first utilized in speech coding [1]. Other various perfect reconstruction filter banks for one-dimensional (1D) subband transformation were presented in [2]-[5].

1D subband transformation theory was extended to two-dimensional (2D) case in [6]-[10].

Subband filters typically support two filtering modes: a convolution mode and a lifting mode. In either mode, an input image should be first periodically extended on all input image boundaries for a half-length of the filter, or the filter itself should be modified at input image boundaries. Convolution-based filters perform a series of multiplica-tions and additions between low-pass and high-pass filter coefficients and extended 2D pixels forming a window matrix in case of non-orthogonal filters, or separate horizontal and vertical processing, in case of orthogonal filters. Lifting-based filtering consists of a sequence of alternative updating of pixels with odd indexes with a weighted sum of pixels with even indexes, and updating of pixels with even indexes with a weighted sum of pixels with odd indexes [11]-[17].

Low-memory subband implementations can also be based on the combination of a fractional wavelet filter [18]-[19] with a low memory image coding system [20], providing a trade-off between smaller RAM size, significantly bigger flash memory size and several times higher number of computations.

II. Subband Transformer

Fig. 1 is a block diagram of state-of-the-art two-dimensional encoder, with $N = 3$ levels of direct two-dimensional (D2D) subband transformers (D2DST).

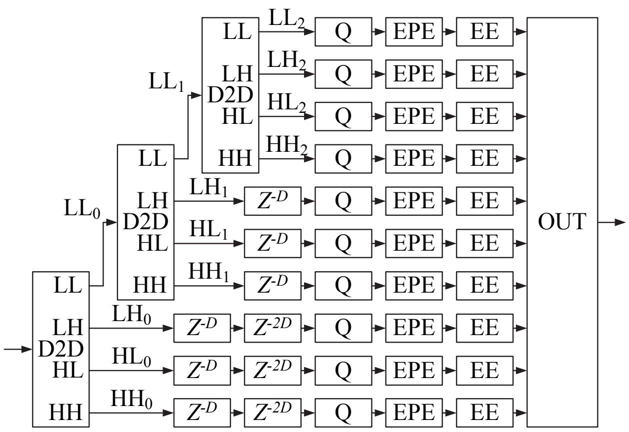

Fig. 1. State-of-the-art two-dimensional encoder.

Uncompressed input pixels are received by the first D2DST, which is applying two separate direct one-dimensional subband transformations (D1DST), first horizontally along the rows and then vertically along the columns. The results of a single-level D2DST are four subbands: LL, LH, HL and HH.

The subband LL corresponds to the low-pass filtering of input pixels and downsampling by two along the rows and the low-pass filtering and downsampling by two along the columns, and simultaneously contains downsampled low frequency horizontal information and downsampled low frequency vertical information.

The subband LH corresponds to the low-pass filtering of input pixels and downsampling by two along the rows and the high-pass filtering and downsampling by two along the columns, and simultaneously contains downsampled low frequency horizontal information and downsampled high frequency vertical information, i.e. horizontal edge information.

The subband HL corresponds to the high-pass filtering

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of input pixels and downsampling by two along the rows and the low-pass filtering and downsampling by two along the columns, and simultaneously contains downsampled high frequency horizontal information, i.e. vertical edge information and downsampled low frequency vertical information.

The subband HH corresponds to the high-pass filtering of input pixels and downsampling by two along the rows and the high-pass filtering and downsampling by two along the columns, and simultaneously contains downsampled high frequency horizontal information and downsampled high frequency vertical information, i.e. diagonal edge information.

State-of-the-art multi-level D2DST is applying single-level D2DST onto an uncompressed input image in the level 0, and then subsequently performing single-level D2DST onto the subband LL_i, produced as a result of previous single-level D2DST. Level 0 subbands LL_0, LH_0, HL_0 and HH_0 are produced by applying D2DST onto the original image. Level 1 subbands LL_1, LH_1, HL_1 and HH_1 are produced by applying D2DST onto the subband LL_0. Level 2 subbands LL_2, LH_2, HL_2 and HH_2 are produced by applying D2DST onto the subband LL_1, etc.

After horizontal D1DST filtering is finished in the level 0, the complete line of transformation coefficients is stored in the memory of size W per each line, wherein W is the width of the image, i.e. the number of pixels within each line. The width of the subband image is reduced by two at each new D2DST level. Therefore, each subband LL_{i+1} at D2DST level i+1 requires memory for half of the lines of the previous subband LL_i at D2DST level i.

State-of-the-art encoders can implement D2DST using finite impulse response (FIR) filters with odd filter length $L = 2 \cdot D + 1$ for convolution filtering using symmetric extension of uncompressed input pixels at input image boundaries. Uncompressed input pixels are periodically received with the period T_P. The total number of received horizontal pixels is $L = D + 1 + D$, due to image extension after delay of $D \cdot T_P$, so the first D2DST filter starts generation of transformation coefficients for LL_0, LH_0, HL_0 and HH_0 subbands using $2 \cdot T_P$ period, due to downsampling by two.

However, linear phase FIR filters require a long processing time and significant memory size. These drawbacks can be eliminated by various lifting filters or a novel type of computationally efficient non-stationary filters, which will be described in the next Section. The memory required for filtering depends on D and the size of stored transformation coefficients according to Table 2, wherein filtering memory size F is provided in Table 1, showing that non-stationary filters require minimum F.

The order of the generation of subband transformation coefficients by the direct subband transformer is almost the opposite to the order expected by the inverse subband transformer, thus requiring additional synchronization memories z^{-kD} in order to compensate different delays at each D2DST level in both the encoder and the decoder. The total synchronization memory size is usually equally split between the encoder and the decoder, using delay lines z^{-D}, $z^{-2 \cdot D}$, etc. However, it is possible to assign the complete synchronization memory to the encoder only or to the decoder only, depending on a particular application. Further description will be devoted only to the symmetrical synchronization memory of equal size in the encoder and the decoder.

TABLE 1: FILTERING MEMORY SIZE *F*.

2DST level	*Filtering memory size F*
Convolution	$2 \cdot D + 1$
Lifting [13]	$D + 2$
Lifting [11]-[12]	$D + 1$
Non-stationary filters	D

TABLE 2: FILTERING MEMORY SIZE PER 2DST LEVEL.

2DST level	*Filtering memory size per 2DST level*
0	$F \cdot W$
1	$\frac{F \cdot W}{2}$
2	$\frac{F \cdot W}{4}$
i	$\frac{F \cdot W}{2^i}$
$N - 1$	$\frac{F \cdot W}{2^{N-1}}$
All levels	$\sum_{i=0}^{N-1} \frac{F \cdot W}{2^i} = 2 \cdot F \cdot W \cdot \left(1 - \frac{1}{2^N}\right)$

All D2DST outputs produce transformation coefficients, which can be quantized into quantized transformation coefficients by quantizers Q, in case of lossy compression, or just passed to encoding probability estimators EPE, in case of mathematically lossless compression. The outputs of encoding probability estimators EPE are symbol probabilities within the specified contexts, which are used by entropy encoders EE, in order to produce an output compressed signal in the output buffer OUT before transmission or storage.

Fig. 2 is a block diagram of state-of-the-art two-dimensional decoder, with $N = 3$ levels of inverse two-dimensional (I2D) subband transformers (I2DST), applying two separate inverse one-dimensional subband transformations (I1DST), horizontally along the rows and vertically along the columns. Both D2DST and I2DST are abbreviated as 2DST.

An input compressed signal is stored in the input buffer IN and after that received by entropy decoders ED. The outputs of entropy decoders ED are received by decoding probability estimators DPE, in order to reconstruct the symbol probabilities within the specified contexts, and feed them back into entropy decoders ED.

The outputs of entropy decoders ED are also received by dequantizers DQ, in order to produce dequantized transformation coefficients, in case of lossy decompression, or just passed to I1DST, in case of mathematically lossless decompression. The output of last I1DST is an output decompressed signal.

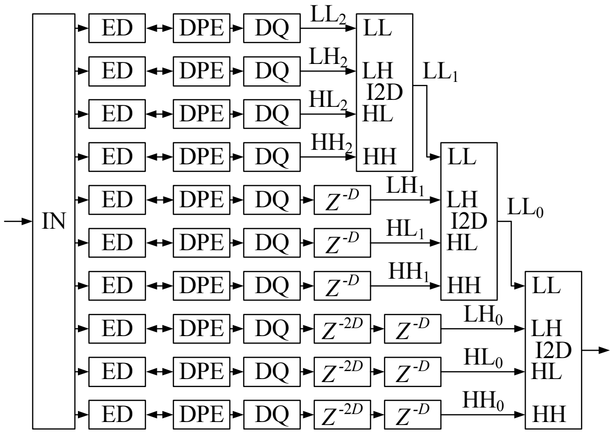


Fig. 2. State-of-the-art two-dimensional decoder.

III. Novel Non-Stationary Filter

State-of-the-art one-dimensional subband transformer used as a building block inside two-dimensional subband transformer provides low-pass filtering and the downsampling by two, in the upper branch in Fig. 3, as well as high-pass filtering followed by the downsampling by two, in the lower branch in Fig. 3. Each second sample in both branches is discarded during this decimation, in spite of memory and processing resources used for their generation.

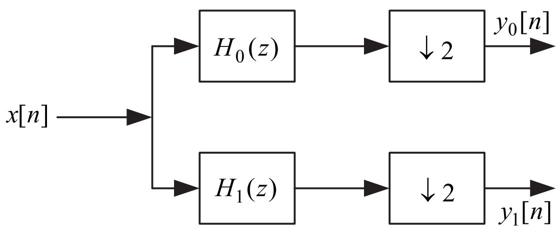


Fig. 3. State-of-the-art direct one-dimensional subband transformer.

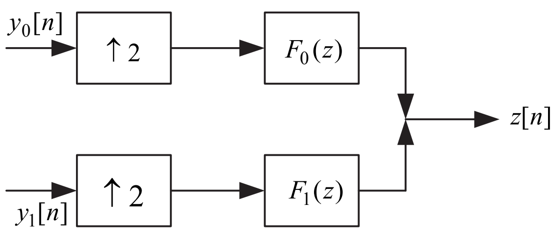


Fig. 4. State-of-the-art inverse one-dimensional subband transformer.

State-of-the-art I1DST provides the upsampling by two and high-pass filtering, in the upper branch in Fig. 4, as well as upsampling by two and low-pass filtering, in the lower branch in Fig. 4. Each second sample in both branches is added during this interpolation, thus increasing memory and processing resources used for their generation.

Novel D1DST and I1DST utilize these wasted memory and processing resources for alternate generation of low-pass and high-pass coefficients, which means that the same non-stationary filter serves as both a low-pass and high-pass filter in subsequent cycles.

Fig. 5 is a block diagram of the direct non-stationary filter for D1DST in two subsequent cycles. The input samples $x[n]$ are received serially, one sample per cycle.

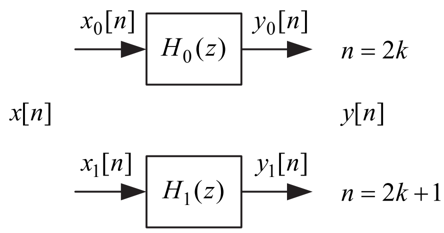


Fig. 5. Novel direct one-dimensional subband transformer in two subsequent cycles.

The input samples $x_0[n]$ with even indexes n are low-pass filtered without any need for downsampling by two, in order to produce output samples $y_0[n]$ (1).

$$y_0[n] = -\frac{1}{8}x[n] + \frac{1}{4}x[n-1] + \frac{3}{4}x[n-2] + \frac{1}{4}x[n-3] - \frac{1}{8}x[n-4] \quad (1)$$

The input samples $x_1[n]$ with odd indexes n are high-pass filtered without any need for downsampling by two, in order to produce output samples $y_1[n]$ (2).

$$y_1[n] = -\frac{1}{2}x[n-1] + x[n-2] - \frac{1}{2}x[n-3] \quad (2)$$

The transfer function of this filter is appropriate to the 5-tap/3-tap analysis (5/3) filter, disclosed in [21], which is used for reversible DWT, providing perfect reconstruction in JPEG2000 standard.

Fig. 6 is a block diagram of the inverse non-stationary filter for I1DST in two subsequent cycles. The output samples $z[n]$ are generated serially, one sample per cycle.

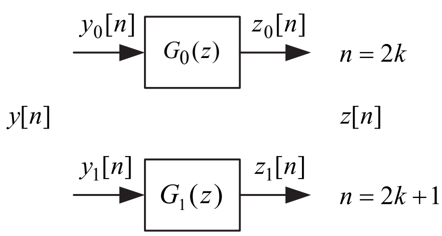


Fig. 6. Novel inverse one-dimensional subband transformer in two subsequent cycles.

The input samples $y_0[n]$ with even indexes n are filtered without any need for upsampling by two, in order to produce output samples $z_0[n]$ (3).

$$z_0[n] = -\frac{1}{4}y[n-1] + y[n-2] - \frac{1}{4}y[n-3] \quad (3)$$

The input samples $y_1[n]$ with odd indexes n are filtered without any need for upsampling by two, in order to produce output samples $z_1[n]$ (4).

$$z_1[n] = -\frac{1}{8}y[n] + \frac{1}{2}y[n-1] + \frac{3}{4}y[n-2] + \frac{1}{2}y[n-3] - \frac{1}{8}y[n-4]. \quad (4)$$

The transfer function of this filter is different from the 5-tap/3-tap synthesis (5/3) filter, disclosed in [21], but also provides perfect reconstruction, for both even index n (5) and odd index n (6), without any final addition operation.

$$z_0[n] = x_0[n-4] \quad (5)$$

$$z_1[n] = x_1[n-4] \quad (6)$$

Multipliers in both direct and inverse non-stationary filters disclosed in this paper can be made either as software shifters or as permanently shifted hardware connections between output and input bit lines, thus removing any necessity for hardware multipliers (Fig. 7).

Transfer functions of novel direct and inverse one-dimensional subband transformers are provided in Figs. 8 and 9, respectively.

It is also possible to implement special non-stationary filter coefficients near image boundaries, instead of symmetric extension of input pixels at image boundaries, which is a well known solution, often used in practical embodiments of subband transformers.

IV. Conclusion

The realization of direct and inverse non-stationary filters is four times simpler than the realization of the equivalent direct and inverse convolution filters, and twice simpler than the realization of the equivalent direct and inverse lifting filters.

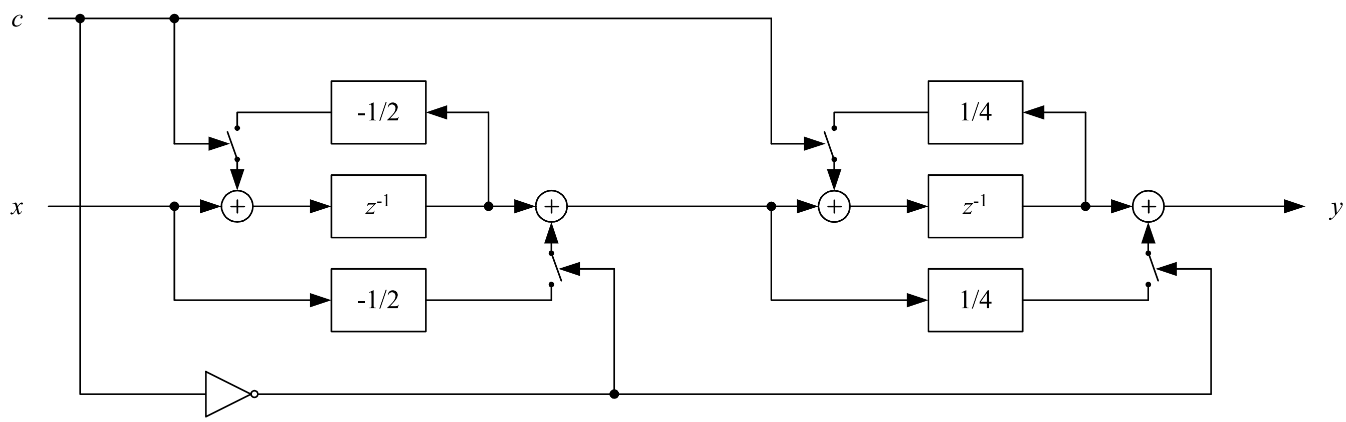
Both direct and inverse non-stationary filters provide alternate low-pass and high-pass filtered samples at its output, with even and odd sample indexes, respectively.

The same filter components are reused for both direct low-pass and high-pass subband filtering, as well as inherent downsampling, thus minimizing memory and processing hardware resources necessary for encoding.

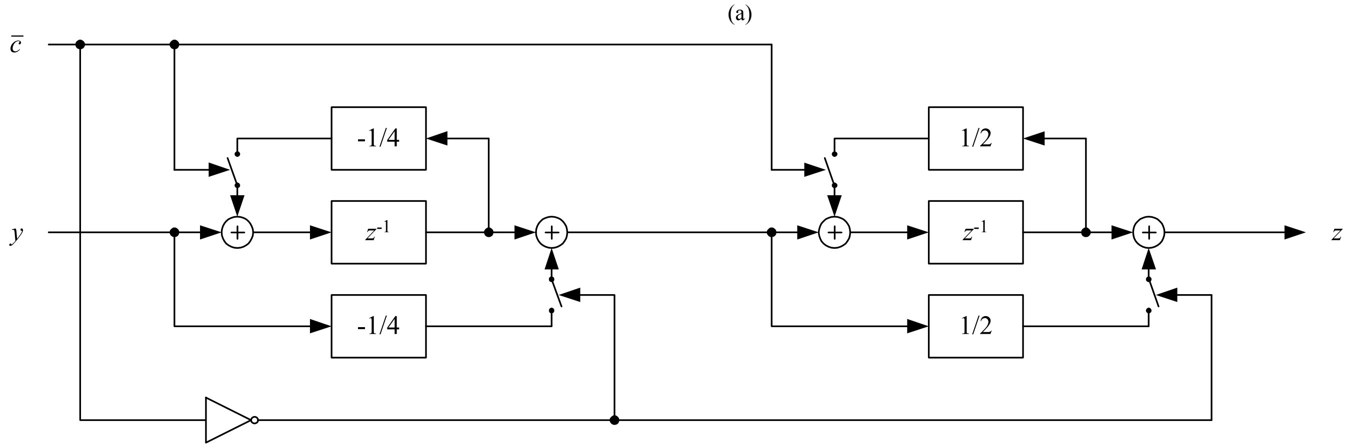
The same filter components are reused for both inverse low-pass and high-pass subband filtering, as well as inherent upsampling, thus minimizing memory and processing hardware resources necessary for decoding.

This solution is compatible with all general purpose microprocessors and digital signal processors, since it does not require any multiplication or division instructions, due to all filter coefficients being the power of two. At the same time, the cache utilization is minimized, thus additionally accelerating the executions speed.

The output of direct filters is easily demultiplexed in the next level, while the input of inverse filters is easily multiplexed by the previous level.



(a)



(b)

Fig. 7. Novel direct (a) and inverse (b) one-dimensional subband transformer.

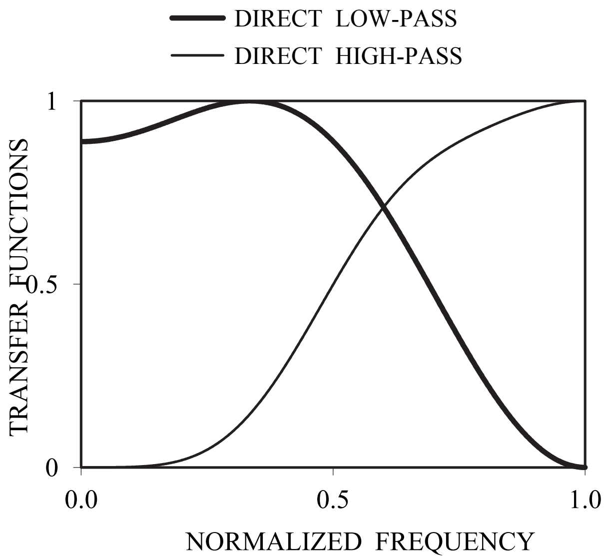


Fig. 8. Direct low-pass (eq. 1) and high-pass (eq. 2) transfer functions of novel direct one-dimensional subband transformer.

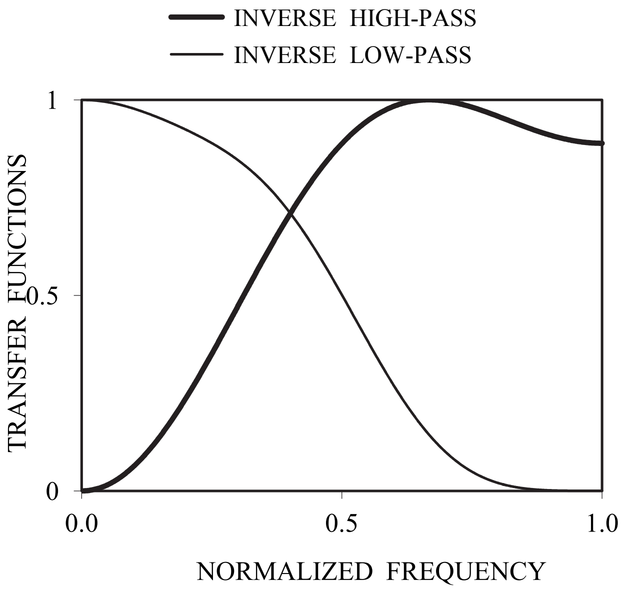


Fig. 9. Inverse low-pass (eq. 4) and high-pass (eq. 3) transfer function of novel inverse one-dimensional subband transformer.

REFERENCES

[1] R. E. Crochiere, S. A. Webber, and J.L. Flanagan, "Digital coding of speech in subbands," *Bell Syst. Tech. J.*, vol. 55, no. 8, pp. 1069-1085, Oct. 1976.

[2] T. A. Ramstad, "Analysis/synthesis filter banks with critical sampling," *Proc. Int. Conf. Digital Signal Processing*, Florence, Italy, Sep. 1984.

[3] M. Vetterli, "Filter banks allowing perfect reconstruction," *Signal Process.*, vol. 10, no. 3, pp. 219-244, Apr. 1986.

[4] M. J. Smith and T. P. Barnwell, III, "Exact reconstruction techniques for tree structured subband coders," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 34, no. 3, pp. 434-441, June 1986.

[5] M. Vetterli and D. Le Gall, "Perfect reconstruction FIR filter banks: Some properties and factorization," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 37, no. 7, pp. 1057-1071, July 1989.

[6] P. J. Burt and E. Adelson, "The Laplacian pyramid as a compact image code," *IEEE Trans. Commun.*, Vol. 31, No. 4, pp. 532-540, Apr. 1983.

[7] M. Vetterli, "Multi-dimensional subband coding: Some theory and algorithms," *Signal Process.*, vol. 6, no. 2, pp. 97-112, Apr. 1984.

[8] J. Woods and S. O'Neil., "Subband coding of images," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 34, no. 5, pp. 1278-1288, Oct. 1986.

[9] M. Antonini, M. Barlaud, P. Mathieu, and I. Daubechies, "Image coding using the wavelet transform," *IEEE Trans. Image Process.*, vol. 1, no. 2, pp. 205-220, Apr. 1992.

[10] A. Zandi, M. Boliek, E. L. Schwartz, M. J. Gormish, an A. Keith, "CREW lossless/lossy medical image compression," Ricoh California Research Center, Menlo Park, CA94025, USA, Sep. 12, 1995.

[11] W. Sweldens, "The lifting scheme: A custom-design construction of biorthogonal wavelets," *Appl. Comput. Harmonic Analysis*, vol. 3, no. 2, pp. 186-200, Apr. 1996.

[12] W. Sweldens, "The lifting scheme: Construction of second generation wavelets," *SIAM J. Math. Anal.*, vol. 29, no. 2, pp. 511-546, 1997.

[13] C. Chrysafis and A. Ortega, "Line based, reduced memory, wavelet image compression," *IEEE Trans. Image Process.*, vol. 9, no. 3, pp. 378-389, Mar. 2000.

[14] Y. Bao and C. Kuo, "Design of wavelet-based image codec in memory-constrained environment," *IEEE Trans. Circuits Syst. Video Technol.*, vol. 11, no. 5, pp. 642-650, May 2001.

[15] C.-H. Yang, J.-C. Wang, J.-F. Wang, and C.-W. Chang, "A block-based architecture for lifting scheme discrete wavelet transform," *IEICE Trans. Fundamentals*, vol. E90-A, no. 5, pp. 1062-1071, May 2007.

[16] J. Lin and M. J. T. Smith, "New perspectives and improvements on the symmetric extension filter bank for subband/wavelet image compression," *IEEE Trans. Image Process.*, vol. 17, no.2, Feb. 2008.

[17] M. Vrankic, D. Sesic, and V. Sucic, "Adaptive 2-D wavelet transform based on the lifting scheme with preserved vanishing moments," *IEEE Trans. Image Process.*, vol. 19, no. 8, Aug. 2010.

[18] S. Rein, S. Lehmann, and C. Gühmann, "Fractional wavelet filter for camera sensor node with external flash and extremely little RAM," in *Proc. ACM Mobile Multimedia Commun. Conf.*, July 2008.

[19] S. Rein, S. Lehmann, and C. Gühmann, "Wavelet image two line coder for wireless sensor node with extremely little RAM," in *Proc. IEEE Data Compression Conf.*, Snowbird, UT, Mar. 2009, pp. 252-261.

[20] S. Rein and M. Reisslein, "Performance evaluation of the fractional wavelet filter: A low-memory image wavelet transform for multimedia sensor networks", *Ad Hoc Networks*, 2010, doi:10.1016/j.adhoc. 2010.08.004.

[21] D. Le Gall and A. Tabatabai, "Subband coding of digital images using symmetric short kernel filters and arithmetic coding techniques," in *Proc. Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, New York, NY, Apr. 1988, pp. 761-765.